9-1-2010

Reserve Prices In A Dynamic Auction When Bidders Are Capacity-constrained

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Economics Letters, 2010

Abstract

Allowing for a reserve price in a dynamic auction with capacity-constrained bidders changes the equilibrium in an unexpected way. The distribution of winning bids contains a mass point; several bidder types "bunch" at the reserve price.

JEL CLASSIFICATION: C73, D44

Keywords: Dynamic Auction, Capacity Constraint, Reserve Price

1 Introduction

A frequent observation made by empirical researchers analyzing procurement auctions for road construction work is that construction firms experience capacity constraints, whereby the cost to a firm of adding another contract to its roster increases with its existing capacity utilization (Bajari and Ye, 2003; De Silva et al., 2002, 2003; Jofret-Bonet and Pesendorfer, 2003).[1] Since the cost distribution of the firm that wins the current auction worsens as a result of the capacity constraint, such firms face an intertemporal tradeoff in their profits: higher profits in the current period come at the cost of lower profits in future periods. Strategic forward-looking firms realize that they incur an opportunity cost or option value in the future by winning the current auction. The behavior of bidders in such auction settings has been theoretically analyzed by Grimm (2007), Jeitschko and Wolfstetter (2002), Jofret-Bonet and Pesendorfer (2006), and Saini (2009).

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[1] Here capacity utilization is defined as the ratio of the firm's outstanding work commitments to its size. The idea underlying this effect is that since a firm's capacity tends to stay fixed in the short term, as its work commitments increase, it must augment its capacity by paying overtime wages, renting additional equipment, moving scarce equipment around from site to site, and using less productive (perhaps older) equipment, all of which typically lead to higher unit costs.

The typical structure of equilibrium strategies is one where a bidder's bid equals the bidder's option value plus the amount that it would have bid in a static one-shot auction. Since the option value is just another markup over cost, bids increase with costs. However, none of the existing models consider the case where—as is common practice in procurement auctions—the procurer may impose a reserve price so that bids higher than the reserve price are rejected. Moreover, the structure of the equilibrium in these models does not allow for a reserve price.[2]

In this paper, we solve for the equilibrium bidding strategies of n capacity-constrained bidders in a sequence of two auctions where the procurer imposes a reserve price in the first period auction. Allowing for a reserve price changes the equilibrium in an unexpected way. We find that the equilibrium bidding strategies are no longer strictly increasing over the range of costs for which a bidder wins with a positive probability. More specifically, there exists an interval of a bidder's costs over which it always bids the reserve price, thus winning the auction with positive probability. Therefore, there exists a mass point in the distribution of winning bids at the reserve price. As usual, we also find a cutoff cost level such that firms with costs higher than the cutoff choose to drop out of the first auction by bidding some amount above the reserve price.

While we present our analysis in the context of a procurement auction, our analysis applies to a more general class of sequential auction models. For instance, an intertemporal linkage analogous to our model is possible in art auctions. An art collector interested in acquiring a series of paintings by a particular painter might find that winning one painting increases her valuation in future auctions for paintings from the same series. Our analysis predicts that if the auctioneer imposes a reserve price in this setting, several bidder types will "bunch" at the reserve price. In addition to theorists, this result should also be of interest to researchers interested in recovering the underlying value distributions of bidders from their observed bid distributions in dynamic auction markets.

2 A Model with Reserve Prices

A procurer auctions off two identical projects using a sequence of two first-price procurement auctions. There are n risk-neutral bidders, each of whom wishes to win both contracts. The costs of the bidders are independent private values. In the first auction, all bidders draw their cost for the project from the interval $[\underline{c}, \bar{c}]$ according to a continuously differentiable probability distribution $F(c)$, with density $f(c)$ that is bounded away from 0 over the support. The auctioneer imposes a reserve price of R in the first auction.[3] In each round the bidders simultaneously

[2] Suppose there are two sequential auctions. Starting from symmetry, in the absence of a reserve prices, there is only one possible configuration of asymmetry in the second auction: someone wins the first auction and everyone else loses. However, in the presence of reserve prices, it is also possible that no one wins the first auction. This makes the bidding decision in the first auction more complicated than simply adding the option value to static bids.

[3] The case where the procurer imposes a reserve price in the second auction as well is straightforward, since then the second auction becomes a simple static auction with a reserve price.

submit sealed bids, and the contract is awarded to the lowest bidder. If there is more than one bidder at the lowest bid (which happens in equilibrium at R with positive probability), then the procurer awards the contract to each bidder with equal probability. A bidder can choose to drop out of the first auction by bidding an amount $OUT > R$.

Due to being capacity-constrained, the winning bidder experiences a first order stochastic dominance shift in its cost distribution on winning the first auction. As a result, the second auction becomes a one-shot asymmetric auction with one 'weak,' and $n-1$ identical 'strong' bidders. Standard results from the theory of static asymmetric auctions allow us to make the following abstraction regarding the profits in the second period auction (Maskin and Riley, 2000): if there is a winner in the first auction, then it gets a payoff of π_w in the second auction, and the remaining $n-1$ bidders get a payoff of π_l in the second auction, where $\pi_w < \pi_l$. If no one wins in the first period auction, all bidders get an expected payoff of π_0 in the second auction, where $\pi_w < \pi_0 < \pi_l$. The idea is that the winning bidder's payoff in the second auction π_w, is lower than what its payoff would have been (π_0) if no one had won the first auction. For a similar reason, the second period payoff of a losing bidder π_l, is greater than π_0. Making this abstraction allows us to avoid the needless complication of describing the straightforward bidding strategies and profit relationships in the second period auction.

We consider the case where the reserve price is binding, that is, $R-(\pi_l-\pi_w) < \bar{c}$. Otherwise, the reserve price does not affect behavior in the auction. We will explain the implication of this assumption below.

3 Equilibrium

Our main result is solving for a Perfect Bayesian Nash Equilibrium (PBNE) of this auction. While the formal statement of the result is summarized in Proposition 1, we now describe the structure of the equilibrium in words. Let $c_{j\neq i}^{\min}$ denote the lowest cost draw among the rivals of bidder i in the first auction. The equilibrium bidding strategy in (1) can be described in three parts.

1. Over the cost interval $[\underline{c}, R-(\pi_l-\pi_w)]$, the players use monotonically increasing strategies. Each cost type's bid is the sum of two components. The first component is the expected minimum of $R-(\pi_l-\pi_w)$ and $c_{j\neq i}^{\min}$ conditional on $c_{j\neq i}^{\min}$ being higher than the bidder's cost type. This is a standard static bid. The second component $(\pi_l-\pi_w)$ is the option value of the bidder in the case where it is the winner in the first auction.

2. Interestingly, all cost types in the interval $[R-(\pi_l-\pi_w), c^*]$ bid the reserve price R; here c^* is as defined in Proposition 1. Thus, there is a mass point in the distribution of equilibrium bids at R. Another interesting feature of bidding over this cost range is that the bidders bid R in spite of the fact that by doing so they will not fully recover $c+(\pi_l-\pi_w)$, their cost draw plus the option value contingent on winning. This is so because with positive

probability, the profit of the losing bidder is π_0 instead of π_l. Since $\pi_0 < \pi_l$, the expected payoff upon losing the auction is lower.

3. All cost types higher than c^* drop out of the first auction by bidding $OUT > R$.

Proposition 1 *Each bidder bidding according to the following symmetric bidding strategy in the first auction constitutes a PBNE of the two-period auction.*

$$b(c) = \begin{cases} \mathbb{E}[\min\{R-(\pi_l-\pi_w), c^{\min}_{j\neq i}\}|c < c^{\min}_{j\neq i}] + (\pi_l - \pi_w) & \text{for } c \in [\underline{c}, R-(\pi_l-\pi_w)] \\ R & \text{for } c \in [R-(\pi_l-\pi_w), c^*] \\ OUT & \text{for } c \in [c^*, \bar{c}]. \end{cases} \quad (1)$$

where c^* solves

$$(R-c^*-(\pi_l-\pi_w))\sum_{k=0}^{n-1}\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k} = (\pi_0-\pi_l)(1-F(c^*))^{n-1}.$$

Proof. We will show that the bidding strategy in (1) is optimal for a bidder given that the other bidders are playing the same strategy.

We will first write down an expression for a bidder's profits upon following this strategy. Since all of them bid R, the winning probability of cost types in the interval $[R-(\pi_l-\pi_w), c^*]$ is given by:

$$P_{win}(R;c^*) = \sum_{k=0}^{n-1}\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k}.$$

Here, $\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k}$ is the probability that k of the other $n-1$ bidders drop out of the first auction, and the remaining $n-k-1$ bidders draw a cost in the interval $[R-(\pi_l-\pi_w), c^*]$, causing them to bid R, which causes the procurer to award the project to each of the $n-k$ bidders, who bid R, with probability $\frac{1}{n-k}$ (tie-breaks). Given the strategies in (1), the expected profit of a bidder that gets a cost draw of c, and bids $b(c)$, is given by:

$$\Pi(b(c);c) = \begin{cases} (b(c)-c+\pi_w)(1-F(c))^{n-1} + \pi_l(1-(1-F(c))^{n-1}) & \text{if } b(c) < R \\ (R-c+\pi_w)P_{win}(R;c^*) + \pi_l(1-P_{win}(R;c^*)) & \text{if } b(c) = R \\ \pi_0(1-F(c^*))^{n-1} + \pi_l(1-(1-F(c^*))^{n-1}) & \text{if } b(c) = OUT > R. \end{cases} \quad (2)$$

If the bidder chooses to participate in the first auction and it bids less than R, then $(1-F(c))^{n-1}$ is the probability that the bidder wins the auction. If the bidder chooses to participate in the first auction with a bid of R, then $P_{win}(R;c^*)$ is the probability that it wins the auction. If the bidder drops out of the auction, then $(1-F(c^*))^{n-1}$ is the probability that every other bidder drops out as well, in which case the bidder gets a profit of π_0. The expression $1-(1-F(c^*))^{n-1}$ represents the probability that at least one of the rival bidders stays in the auction, in which case the bidder's payoff is π_l. The expression in (2) simplifies to:

$$\Pi(b(c);c) = \begin{cases} (b(c) - c - (\pi_l - \pi_w))(1 - F(c))^{n-1} + \pi_l & \text{if } b(c) < R \\ (R - c - (\pi_l - \pi_w))P_{win}(R;c^*) + \pi_l & \text{if } b(c) = R \\ (\pi_0 - \pi_l)(1 - F(c^*))^{n-1} + \pi_l & \text{if } b(c) = OUT > R. \end{cases} \tag{3}$$

We now show that c^* exists and lies in $(R - (\pi_l - \pi_w), \overline{c})$. At a given cost draw, a bidder makes the stay-in-or-drop-out decision by comparing the profits from staying in, and bidding an amount less than or equal to R, to the profits from dropping out. The maximum cost at which a player will make a positive payoff by placing a winning bid in the first auction is $R - (\pi_l - \pi_w)$, where it bids R. Note that we assumed that $R - (\pi_l - \pi_w) < \bar{c}$. Otherwise, the reserve price is not binding. Now since $(\pi_0 - \pi_l)(1 - F(c^*))^{n-1} < 0$, then for some cost draws higher than $R - (\pi_l - \pi_w)$, it might make sense for the bidder to stay in and place a bid of R in the first auction, even when doing so means that $R - c - (\pi_l - \pi_w) < 0$. That is, even when it will not entirely recover its cost plus option value conditional on winning. The bidder should drop out of the first auction only for cost draws higher than c^*, which is defined as the cost level at which the bidder is indifferent between staying in, and bidding R, and dropping out. Thus, c^* solves:

$$(R - c^* - (\pi_l - \pi_w))P_{win}(R;c^*) = (\pi_0 - \pi_l)(1 - F(c^*))^{n-1}.$$

In order to see that c^* exists, define the following functions for $c \in [R - (\pi_l - \pi_w), \overline{c}]$:

$$\begin{aligned} A(c) &\equiv (R - c - (\pi_l - \pi_w))P_{win}(R;c), \\ B(c) &\equiv (\pi_0 - \pi_l)(1 - F(c))^{n-1}. \end{aligned}$$

By assumption, this interval is non-empty. Moreover, the following properties of the functions can be easily verified: $A(R-(\pi_l-\pi_w)) = 0$; $A(\overline{c}) < 0$; $A(c),\ A'(c) < 0$;[4] and $B(R-(\pi_l-\pi_w)) < 0$; $B(\overline{c}) = 0$; $B(c),\ B'(c) > 0$. Since $A(\cdot)$ and $B(\cdot)$ are continuous functions, these properties imply that the two functions intersect in the interior of the interval $[R - (\pi_l - \pi_w), \overline{c}]$. This proves the existence of c^*. We now know that $c^* \in (R - (\pi_l - \pi_w), \overline{c})$, and that $c \lesseqgtr c^* \leftrightarrow A(c) \gtreqless B(c)$ for $c \in [R - (\pi_l - \pi_w), \overline{c}]$.

We now show that the strategy in (1) is optimal given that the other $n-1$ players are playing this strategy.

First we will consider the case where $c \in [\underline{c}, R - (\pi_l - \pi_w)]$. We will begin by showing that the problem of each cost type in this range is identical to the problem of each cost type over the same range in a special one-shot game. Next, we will write down this player's optimal strategy in the one-shot game, which must then also be optimal in our game. The optimal bidding strategy in the one-shot game will turn out to be the same as the one depicted for this cost range in

[4] Note that $A'(c) < 0$ because an increase in c^* means that the rivals of a bidder are less likely to drop out and more likely to compete with the bidder at the reserve price, thus lowering its profits upon staying in and bidding R.

(1). So for the sake of our argument, consider a special one-shot auction game where each of n bidders gets a cost draw of $c + (\pi_l - \pi_w)$ where c is distributed over $[\underline{c}, \bar{c}]$ according to $F(c)$. The auctioneer imposes a reserve price of $R < \bar{c} + \pi_l - \pi_w$. Also, it pays each bidder the amount π_l irrespective of the auction's outcome. If a bidder with cost c submits the lowest bid b_{min}, it wins the auction and gets an additional payoff of $b_{\text{min}} - c - (\pi_l - \pi_w)$. Now suppose each bidder is playing the strategy from Proposition 1 in this one-shot game. Then a bidder's expected profits in the one-shot game are:

$$\pi(b(c); c) = \begin{cases} (b(c) - c - (\pi_l - \pi_w))(1 - F(c))^{n-1} + \pi_l & \text{for } b(c) \leq R \\ \pi_l & \text{for } b(c) = OUT > R. \end{cases}$$

Notice that over the cost range $[\underline{c}, R - (\pi_l - \pi_w)]$, the profits in the one-shot game are the same as the profits in our model. While the winning probability at a cost of $R - (\pi_l - \pi_w)$, where bidders in both models bid R, is lower in our model,[5] the *profits* at $R - (\pi_l - \pi_w)$ equal π_l in both models. Using standard auction theory arguments, made in the appendix, it can be shown that the symmetric equilibrium strategy in the one-shot game is:

$$\hat{b}(c) = \begin{cases} \mathbb{E}[\min\{R - (\pi_l - \pi_w), c^{\min}_{j \neq i}\} | c < c^{\min}_{j \neq i}] + (\pi_l - \pi_w) & \text{for } c \in [\underline{c}, R - (\pi_l - \pi_w)] \\ OUT & \text{for } c \in [R - (\pi_l - \pi_w), \bar{c}]. \end{cases} \tag{4}$$

Note that the profits from choosing OUT in our model, $(\pi_0 - \pi_l)(1 - F(c^*))^{n-1} + \pi_l$, are lower than the profits from choosing OUT in the one-shot game (π_l). Thus, any cost type in $[\underline{c}, R - (\pi_l - \pi_w)]$ that finds it optimal to stay in and bid $b(c)$ in the one-shot game will find it optimal to stay in and bid $b(c)$ in our model as well. Thus, the bidding behavior over the cost range $[\underline{c}, R - (\pi_l - \pi_w)]$ in our model is identical to the bidding behavior in the one-shot model given in (4) over the same cost range. For the remainder of our argument, we do not need to use this one-shot game example.

Now consider the case where $c \in [R - (\pi_l - \pi_w), c^*]$. Since $A(c) \geq B(c)$ over this range, a cost type in this range should not choose OUT, since it can have a higher payoff by bidding R. Note that this type would not want to bid less than R because the highest type in the interval $[\underline{c}, R - (\pi_l - \pi_w)]$ finds it profit-maximizing to bid R. Then, a higher type in $[R - (\pi_l - \pi_w), c^*]$ can not find it profit-maximizing to bid less than R.

Finally, consider the case where $c \in [c^*, \underline{c}]$. Since $A(c) \leq B(c)$ over this range, a cost type in this range will find it profitable to drop out of the first auction by bidding OUT rather than staying in and bidding R. While we have shown that these types will not bid R, the argument for why they will not bid less than R is the same as in the previous paragraph.

Thus, we have shown that for every cost draw in the interval $[\underline{c}, \bar{c}]$, the bidding stratgey in Proposition 1 is optimal for a bidder when the other bidders are using the same strategy.

[5] Note that $[1 - F(R - (\pi_l - \pi_w))]^{n-1} > \sum_{k=0}^{n-1} \frac{1}{n-k} C_k^{n-1} [1 - F(c^*)]^k [F(c^*) - F(R - (\pi_l - \pi_w))]^{n-1-k}$. Intuitively, in the one-shot example, only cost types equal to $R - (\pi_l - \pi_w)$ bid R, while in our model, all cost types in $[R - (\pi_l - \pi_w), c^*]$ bid R, thereby intensifying competition at the reserve price bids.

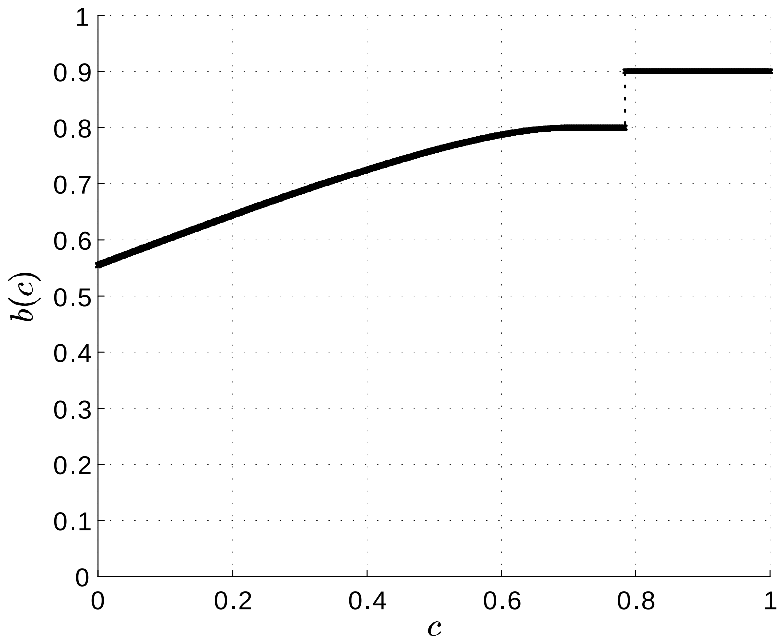

Figure 1: Uniform Distribution

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3.1 Example: Uniform Distribution

We now illustrate the equilibrium bidding strategies in the case with $n = 2$ bidders where each bidder's cost is uniformly distributed over $[0, 1]$. Suppose $R = 0.8$, $\pi_l - \pi_w = 0.1$, and $\pi_0 - \pi_l = -0.1$. Also, a player can drop out of the first auction by bidding $OUT = 0.9$. For these parameter values, we calculated $c^* = 0.7838$. From Proposition 1, the bidding strategy in the first auction is:

$$b(c) = \begin{cases} c + \frac{\int_c^{0.7}[1-c]dx}{1-c} + 0.1 & \text{for } c \in [\underline{c}, 0.7] \\ 0.8 & \text{for } c \in [0.7, 0.7838] \\ OUT & \text{for } c \in [0.7838, 1]. \end{cases}$$

This strategy is depicted in Figure 1. The bids increase with cost till $c = 0.7$, after which the bidder simply bids the reserve price as long as its cost is less than $c^* = 0.7838$. For higher cost draws, the bidder drops out of the first auction. The fact that cost types in the interval $[0.7, 0.7838]$ bunch at R will produce a discontinuous jump in the cdf of winning bids at R.

4 Concluding Remarks

Motivated by markets in which the same set of bidders repeatedly competes over a series of dynamically-linked auctions, this paper asks what the effect of imposing reserve prices on bidder behavior is in such settings. While bids monotonically increase with costs when the reserve price is non-binding, with binding reserve prices the equilibrium changes to one where several cost types submit the reserve price. Therefore, the distribution of winning bids contains a mass point at the reserve price. In addition to contributing to a theoretical understanding of bidder behavior in dynamic auction markets, this finding should also be useful for empirical work that seeks to infer the underlying values of bidders from their observed bids.

Acknowledgement

This paper is based on Chapter 4 of my dissertation completed at Johns Hopkins University. I would like to thank Joe Harrington for helpful and encouraging comments.

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Appendix

The special one-shot game described in the proof of Proposition 1 is strategically equivalent to the following one-shot game. Consider an auction where each of n bidders draws its cost from the interval $[\underline{c}, \bar{c}]$ according to the distribution $F(c)$. The procurer imposes a reserve price of R. We show that the equilibrium bidding strategy in this auction is given by:

$$\begin{aligned} \beta(c) &= \mathbb{E}[\min\{R, c_{j \neq i}^{\min}\} | c < c_{j \neq i}^{\min}], \\ &= c + \frac{\int_c^R [1 - F(x)]^{n-1} dx}{[1 - F(c)]^{n-1}}, \end{aligned} \tag{A.1}$$

for $c \in [\underline{c}, R]$, and $\beta(c) = OUT$ for $c \in (R, \bar{c}]$. We follow the exposition in Krishna (2002) for a regular auction. For cost types $c \in [R, \bar{c}]$, winning leads to a negative payoff, since the maximum payment is R, while choosing OUT leads to a payoff of 0. Therefore, choosing OUT is in fact a dominant strategy for them. We now show that for cost types $c \in [\underline{c}, R]$, the strategy (A.1) is a best response for a player when its rivals are using the same strategy. Denote by $\Pi(z; c)$ the profit of a player when its cost is c, but it bids as if its cost is $z \neq x$. We will show that $\Pi(z; c) - \Pi(c; c) < 0$ for $z \neq x$.

$$\begin{aligned} \Pi(z; c) &= [1 - F(z)]^{n-1}(\beta(z) - c) \\ &= [1 - F(z)]^{n-1} \left(z + \frac{\int_z^R [1 - F(x)]^{n-1} dx}{[1 - F(z)]^{n-1}} - c \right) \\ &= [1 - F(z)]^{n-1}(z - c) + \int_z^R [1 - F(x)]^{n-1} dx. \\ \Pi(c; c) &= [1 - F(c)]^{n-1}(\beta(c) - c) \\ &= \int_c^R [1 - F(x)]^{n-1} dx. \\ \Pi(z; c) - \Pi(c; c) &= [1 - F(z)]^{n-1}(z - c) - \int_c^z [1 - F(x)]^{n-1} dx < 0. \end{aligned}$$