2021

The Mathematics of Mutual Aid: Robust Welfare Guarantees for Decentralized Financial Organizations

Christian Ikeokwu

Oberlin College

The Mathematics of Mutual Aid: Robust Welfare Guarantees for Decentralized Financial Organizations

CHRISTIAN IKEOKWU, Oberlin College, Ohio

Mutual aid groups often serve as informal financial organizations that don't rely on any central authority or legal framework to resolve disputes. Rotating savings and credit associations (roscas) are informal financial organizations common in settings where communities have reduced access to formal financial institutions. In a rosca, a fixed group of participants regularly contribute small sums of money to a pool. This pool is then allocated periodically typically using lotteries or auction mechanisms. Roscas are empirically well-studied in the development economics literature. Due to their dynamic nature, however, roscas have proven challenging to examine theoretically. Theoretical analyses within economics have made strong assumptions about features such as the number or homogeneity of participants, the information they possess, their value for saving across time, or the number of rounds. This work presents an algorithmic study of roscas. We use techniques from the price of anarchy in auctions to characterize their welfare properties under less restrictive assumptions than previous work. We also give a comprehensive theoretical study of the various Rosca formats. Using the smoothness framework of Syrgkanis and Tardos [46] and other techniques we show that the most common rosca formats have welfare within a constant factor of the best possible. This evidence further rationalizes these organizations' prevalence as a vehicle for mutual aid. Roscas present many further questions where algorithmic game theory may be helpful; we discuss several promising directions.

1 INTRODUCTION

1.1 Mutual Aid and Informal Financial Organizations

This map shows the number of adults without access to a bank account or formal credit in each country.

Fig. 1. Source: Map O.3 in Demirguc-Kunt et al. [24]

As you can see a large percentage of low- and middle-income countries have high numbers of people without formal credit, But as shown on the map this isn't a problem exclusive to low- and middle-income nations. Globally over 1.7B people don't have access to any formal financial institutions. In the U.S alone, over 7.1M adults don't have a bank account [21] and over 45M don't have a credit score [18]. When people aren't able to access such vehicles, they have difficulty saving for purchases and investments and don't have a robust safety net to deal with negative unforeseen circumstances. Unfortunately, the people who are most unable to deal with unforeseen shocks are most likely to experience them [1]. As we've seen during the COVID-19 pandemic, such shocks can have devastating consequences and disproportionately affects ethnic/racial minorities and other economically vulnerable groups. For example, before coronavirus, 33% of White households in North Carolina households would have been unable to cover all of their expenses for three months without assistance while this number was 63% and 68% for Black and Latinx households respectively [30].

Mutual aid groups often serve as a way for people to self-organize to address these issues. Mutual aid groups go beyond just financial help and take a specialized local approach to supporting communities. As a result, they are more effective because small mutual aid groups are able to act quickly and provide targeted support without dealing with incentive issues like corruption or moral hazard that plague centralized approaches. Mutual aid is also distinguished from centralized resource distribution by its focus on reciprocity, horizontality, and equality. Mutual aid groups remove the divide between helper and helped and can effectively identify assess whether a person lacks sufficient resources. In this way, they not only reach out to people left behind by centralized relief programs but also go beyond crisis relief and offer community empowerment. Jun and Lance [31] show that such grassroots measures have been surprisingly effective at responding to the pandemic. In fact, the global pandemic has seen a rise of such mutual aid vehicles in the western nations see [20] and [47]. Mesch et al. [39] reports that 21% of U.S. households indicated that their giving to charitable organizations focused on purposes besides basic needs/health and religion (e.g., education, arts, the environment) decreased during the COVID-19 pandemic, instead U.S. households prioritized giving to meet the pressing needs of those in their area.

Mutual aid often takes shape in the form of *informal financial organizations* that don't rely on any legal framework or central authority to resolve disputes or enforce compliance. They are often used by the developing world, the economically vulnerable and immigrant and refugee communities see [6] and [38]. In addition to the social support role these typically play informal financial organizations are used for (1) *Informal Insurance* like in risk-sharing networks (see Fafchamps and Lund [26] for an example), (2) *Informal Savings* like in Accumulating Savings and Credit Associations (see [11]) and (3) *Informal Credit* like in lending clubs (see [22].) But with such simple structures, no central enforcer or legal framework how effective can these informal financial organizations be at allocating resources to those with the most need? In this paper, we investigate this question for the *Rotating Savings and Credit Associations (roscas)* an incredibly widespread mutual aid organization and show that it is effective at allocating resources to those with the most need.

1.2 Rotating Savings and Credit Associations

Rotating savings and credit associations (roscas) are informal financial organizations that are commonly used in low- and middle-income nations as well as many immigrant and refugee populations. These institutions serve as one mechanism for saving, credit, and insurance – among other forms of financial and social support – in settings where communities have reduced access to centralized financial institutions. Roscas often function as follows: a fixed number of participants, usually from similar socio-economic backgrounds, come together to contribute small sums of money to a pot in a periodic manner. At each meeting, the pot is then allocated to one participant who has not yet received it. The allocation is usually determined by lottery or auction but roscas, but flexibility is innate to roscas and so there are many variations on this format see Ardener [6]. Once a full round is completed and each participant has received a pot exactly once, the rosca may disband or start over for another full round. Recipients of the pot often use their influx of cash to invest, especially in more expensive durable goods such as farm equipment, appliances, or vehicles. As a result, this simple setup enables participants to effectively perform peer-to-peer lending: members who receive the pot earlier borrow from those who receive it later.

The prevalence of roscas cannot be overstated. They have been observed as endemic to communities on five continents; [16] alone documents roscas in 85 countries. [17] estimates that roscas account for about one-half of Cameroon's national savings, and [8] estimates that over 1/6 of households in Ethiopia's highlands participate in *ekub* – the region's variant of roscas. Roscas are also vital fixtures in immigrant populations, enabling members to find community and financial resources where such things might not otherwise be readily available.

The distinguishing characteristic of the rosca according to Ardener [6] is the *recurrent rotation of funds*. The rosca allocation mechanism has often been used as a classification criteria in both the economic and anthropological literature Kovsted and Lyk-Jensen [37]. Table 1 describes the different rosca categories in each column. The name of the Rosca category is in the first row, the method for determining allocation order is in the second row and examples are described in the subsequent rows.

Fixed Rosca	**Random Rosca**	**Bidding Rosca**	**Market Rosca**
Fixed criterion	Lottery	Contribution amount	Negotiation
Wealth Level [36]	*Upfront lottery random rosca [13]*: lottery for receipt dates is held before the first meeting	*Premium bidding rosca [37]*: unallocated members bid to receive the pot that meeting by promising higher future contributions to the pot	*Random roscas with aftermarket [38]*: members are given initial random assignments and are allowed to switch positions through bilateral agreements.
Social Status [6]	*Sequential lottery random rosca [37]*: the lottery is held at the beginning of each meeting	*Discount bidding rosca [37]*: the bids are discounts to the contributions to the pot from the other members	*Divisible rosca. [6]*: members are allowed to own fractions or multiple shares in the rosca and members can split shares
Trust/Risk [38]		*Upfront bidding rosca [13]*: the bids to determine receipt dates is held before the first meeting	

Table 1. Rosca Taxonomy

Roscas' ubiquity has prompted nearly three decades of study by economists, starting with Besley et al. [13]. Economic theory in particular has sought to explain the way roscas function as insurance, saving, and most importantly, lending devices for members. Theoretical studies of roscas have proved challenging for three key reasons. First, roscas are dynamic: participants make decisions in an online fashion, and condition their future decisions on past outcomes. Second, as participants are often uncertain about each others' financial needs, a theory of roscas must accommodate incomplete information. Finally, the sums of money in play in roscas are significant enough that agents' utilities are nonlinear. The standard economic approach of solving for auction equilibrium under any one of these phenomena in isolation is already challenging. More comprehensive approaches have only succeeded with aggressive simplifying assumptions.

This work initiates an algorithmic study of roscas. In particular, the theory of worst-case analysis of games, or *price of anarchy* provides tools for studying welfare properties of equilibria without directly solving for them. Using these tools, we investigate the allocative efficiency of roscas i.e how well roscas do at allocating resources to the members with the most need. We ask how well these institutions coordinate saving and lending to people whose opportunities to invest a rosca's pot may be heterogeneous across individuals and across time. We show that under a wide range of assumptions on both people's values and the rules of the pot allocation protocol itself, roscas are able coordinate a groups' lending and borrowing in a way which approximately maximizes the group's total utility. In this way, we provide a flexible analysis to match and explain the diversity of circumstances where roscas appear.

1.3 Our Contributions

To model saving and lending in roscas, we generalize the formulation of [14]. We assume each agent seeks to purchase an investment such as a durable good, but only has means to do so upon winning a rosca's pot. We study the price of anarchy of a lottery, market-based and auction-based roscas, where during each meeting, agents bid or make trades from initial assignments to choose a winner among those who have not yet received a pot. Agents must weigh the value of investing earlier against the utility-loss from spending their income to win. In an earlier version of this work, on which we gave a spotlight talk at the *NeurIPS 2020 Workshop on Fair AI in Finance* and a short talk at the *IJCAI 2021 Workshop on AI for Social Good (AI4SG)* we gave two main results:

- We showed that equilibria of the two most prevalent auction-based rosca formats, ascending- and descending-price auctions (Def 4.1) maximize the total utility of participants up to a constant factor which degrades smoothly as peoples' marginal values for money grows.
- We showed that any rosca that uses a "reasonable" pot allocation protocol each period is guaranteed high equilibrium welfare. In other words, we gave sufficient conditions on single-item auctions which guarantee that the rosca with that auction protocol has good welfare i.e low price of anarchy (Def 3.3).

We proved these results using a variant of the smoothness framework developed in [46]. The rosca setting required us to adapt the framework to three challenges. First, payments in each stage of a rosca are typically redistributed among members, complicating the standard accounting. Second, participants' utilities depend on their payments (and rebates) in a nonlinear way. Finally, the sequential nature of roscas doesn't lend itself to standard smoothness composition arguments. More comprehensive approaches at rosca analysis had only succeeded with aggressive simplifying assumptions prior to that version. This version presents never before seen analysis and results, that expands on the initial work in two significant ways:

- We go beyond auction based Roscas and show that most of the commonly observed Rosca formats, inn general maximize the total utility of participants up to a constant factor. We give this bounds for random, bidding and market roscas.
- We greatly expand the criteria for "reasonable" pot allocation protocol to include weakly smooth mechanisms which include many canonical protocols like the second-price or vickrey auction (Def. 4.4) and rederive all of our previous results with fewer assumptions.

The expanded theory for weakly smooth mechanism for bidding roscas and the addition of the theoretical analysis of market and random roscas to our initial analysis make this the first comprehensive algorithmic study of Roscas.

2 RELATED WORK

Extensive empirical evidence shows the pervasiveness and positive impact of these financial institutions and their efficacy at improving economic, health, and social outcomes. In addition to works already mentioned, Raccanello and Anand [43] document the use of roscas to finance high healthcare expenditures and build wealth in Mexico. Aredo [8] demonstrates the dynamic and flexible nature of roscas in Ethiopia and documents the wide variations on rosca that exist. Pasha and Dayrra [42] show that ekub are a big engine of small business finance in Arba Minch and private businesses actually prefer raising money from roscas than from formal financial organizations. Amankwah et al. [4] and Alabi et al. [3] study roscas in Ghana, Ogujiuba et al. [41] in Nigeria, and Kabuya [32] in Eswatini. Alabi et al. [3] show that people joined roscas for their perceived efficiency in Ghana and show that roscas increased the development of micro and small enterprises.

Many studies analyze composition and participation differences across age, ethic, gender, and socioeconomic lines. Adams Dale and Canavesi [2] and Ardener and Burman [7] show that rosca participation is higher among women than men. Anderson and Baland [5] shows that employed married women in Kenya at times use roscas as a way to take their income and save it in order to protect their earnings from their husbands who may want to spend it immediately. Roscas are known to often include members from similar socio-economic backgrounds, in part as a form of insurance [8]. Nonetheless, Klonner [36] shows that intragroup diversity is associated with higher rates of bidder altruism and more efficient intra-rosca allocations.

Economists have also studied the interaction of roscas with formal credit markets. For instance, Besley et al. [14] show that while credit markets are more efficient than roscas, there are situations in which one can expect a higher ex ante expected utility in roscas than formal credit markets. Relatedly, Fang et al. [27] show that in cases where formal credit markets are present but imperfect, roscas and credit markets can complement one another, thereby improving social welfare.

A different line of work studies roscas as a form of insurance. Klonner [34] develops the first such model of roscas, comparing their performance to risk-sharing contracts. In this model, roscas can serve as a form of financial intermediary and generate more returns for more risk-averse participants who function as a sort of insurance provider. This work also analyzes the risk-sharing performance of a simple set-up where a group of homogeneous people run several bidding roscas simultaneously. He shows that this set-up performs as well as a linear risk-sharing contract and is more enforceable since it carries fixed rather than variable contributions from the participants. Similarly, Baland et al. [9] shows some previously-observed behaviors that may be deemed irrational can be explained by considering roscas as a form of insurance. Calomiris and Rajaraman [19] empirically demonstrate that roscas are used as a form of insurance. Using data from roscas in India, Calomiris and Rajaraman [19] show the unpredictable needs for funds, is reflected by the volatility of interest rates implicit in winning bids.

This work is most closely related to the micro-economic work on roscas. Besley et al. [13, 14] lay the first theoretical model of roscas and subsequent studies focus on providing comparative welfare guarantees, e.g., between random and bidding roscas, or between roscas and alternative institutions. They make stronger asssumptions than us like homogeneity across participants and participants values being homogenous across time. They(Besley et al. [13, 14]) show that both the random and bidding rosca are inefficient but do not give bounds on this inefficiency like we do. Kovsted and Lyk-Jensen [37] analyze differences between random and bidding roscas, again under the assumption that people are saving for a large purchase. They allow for some heterogeneity in people's access to credit, Kovsted and Lyk-Jensen [37] again provide a comparative welfare analysis between bidding and random roscas. They do not, however, quantify the welfare differences or loss as we do.

This work applies techniques from auction theory and the price of anarchy literature.Smoothness is the main tool used to analayze roscas. Syrgkanis and Tardos [46] adapts the theory of smooth games from Roughgarden [44] to auctions. They give a sufficient condition for a game to have approximately-optimal equilibrium welfare, and show that smoothness is preserved in combination with other smooth mechanisms. In addition, smoothness-derived guarantees generalize beyond standard quasilinear, full-information settings to learning outcomes and Bayes-Nash equilibria, large games Feldman et al. [28], risk-averse agents, Kesselheim and Kodric [33], and more. Roughgarden et al. [45] give a complete survey.

3 WARMUP: RANDOM ROSCAS WITH AFTER-MARKET

We first analyze a simpler model of roscas to illustrate the kind of welfare analysis we will do on the more analytically challenging auction based roscas. Ardener [6] reports that rosca in which participants use a lottery to determine the receipt order i.e random roscas are the most common rosca format. However as observed in Mequanent [38] often times the lottery is used just as an initial assignment. Rosca members are free to trade places with each other later, such trades are usually induced via side payments or as a result of altruism [38]. In this section we model these *Random Roscas with After-Market* (RRAM) and show that they are a 2-approximation of the optimal assignment.

We first give a description of the general rosca process. Formally, a rosca for n people takes place over n discrete time periods. Each period, three things occur. (1) Each person pays an amount p_0 into a common pot. (2) A winner for the np_0 units of money in the pot is decided among those who have not yet won. (3) People make any payments p_i^t induced by the selection process in period t. This could be positive or negative depending on the selection process and the person's outcomes.

ALGORITHM 1: General Rosca Process

input : A set members of members and an allocation procedure M

output : An n-dimensional array assignment where assignment$[t]$ is the person who receives the pot in period t. An $n \times n$-dimensional payments where payments$[t][i]$ give i's payments above p_0 in period t

```
active ← members;
assignment;
payments;
for t ← 1 to n do
    pot ← 0;
    for i ← 1 to n do
        pot ← pot + members[i].pay(p0);
    end
    w ← M.Allocate(active, t, pot);
    payments[t] ← M.ChargePayments(active, t, pot);
    assignment[t] ← w;
    Remove(active, w);
end
return assignment, payments;
```

The payments p_0 into the pot are decided ex ante, during the formation of the rosca. We will not model the process of selecting p_0, instead taking p_0 as given, See [13], [14], [34] for models where p_0 is endogenous people can optimize over the choice of p_0. As in previous models like [13], [14], we do not think about rosca defaults and assume everyone pays their contribution. Empirical evidence suggests this is reasonable as the personal cost of defaulting is high in informal financial organizations because these groups usually serve a social support role and rely on interpersonal relationship. This social structure has been used in the nobel winning work of Banerjee and Duflo [10] and by Besley and Coate [12] to improve default rates and credit access in rural impoverished communities.

Roscas give individuals without access to banking a way to save for substantial investments such as large durable goods. We assume each person desires to pay for a one-time investment, yielding for person i a utility stream of ξ_i^t units at period t if and only if the person has invested in the past. This utility stream could be things like daily profit from a taxi to monthly crops from a farm. We follow [13] and assume the cost of the investment is homogeneous across people, and equal to the pot amount np_0. However, we allow ξ_i^t to vary across time and across people. In this section while we are modelling a simple Rram, we assume people have *quasilinear utilities* i.e utility functions that are linear in one argument. In our case, utilities are linear in wealth consumption, in Section 5 we generalize this to concave utilities. Each period t, each person has initial wealth level w_i^t. In a Rram after paying p_0 into the pot and making additional side payments p_i^t during the after-market, where they make bilateral swap agreements after some initial assignment, the person consumes their remaining wealth for period t for some utility of $w_i^t - p_0 - p_i^t$. Let $\mathbb{I}_i^t$ indicate whether a member has invested by time t. A person's utility at time t is thus $w_i^t - p_0 - p_i^t + \xi_i^t \mathbb{I}_i^t$.

Let $x(i) : [n] \to [n]$ be a bijective map from people to receipt periods, so $x(i) = j$ if person i receives the jth pot. We let $v_i : [n] \to \mathbb{R}_{0+}, v_i(j) = \sum_{t=j}^{n} w_i^t - p_0 - p_i^t + \xi_i^t \mathbb{I}_i^t$ denote person i's value for receiving the pot in period j. We evaluate an assignment by the total utility it generates for all participants i.e *utilitarian social welfare*

Definition 3.1 (Utilitarian Social Welfare). The *Social Welfare* for an assignment x is given by

$$\textsc{welf}(v, x) = \sum_{i=1}^{n} v_i(x(i)) \tag{1}$$

We allow people to make pairwise trades if the trade is beneficial to overall social welfare. This is reasonable, because even if the swap isn't beneficial to one person an increase in social welfare implies that the benefiting stakeholder could simply pay the other one to induce the trade. Thus we say an outcome is *swap-stable* if there are no socially beneficial swaps.

Definition 3.2 (Swap-Stable). An assignment $x : [n] \to [n]$ is *swap-stable* if $\forall i, j$

$$v_i(x(i)) + v_j(x(j)) \geq v_i(x(j)) + v_j(x(i)) \tag{2}$$

We adopt the convention that the optimal assignment OPT(v) is $x(i) = i$ (we can simply relabel people without loss of generality) we compare the *utilitarian social welfare* achieved by the rram to that of the optimal assignment. To evaluate the quality of a protocol we compare the performance of its outcomes (swap-stable) on an objective (social welfare) to that achieved by the optimal algorithm OPT, i.e the algorithm that has perfect information and unlimited computing power and can force everyone to do the optimal action for that objective. We express this quality as a ratio between the worst objective function value of an outcome of our protocol and an optimal outcome. This ratio is analogous to the approximation ratio from algorithm design and is called the *price of anarchy* because it captures the cost to society from the lack of a benevolent central enforcer.

Definition 3.3 (Price of Anarchy).

$$\max_{x \in \mathcal{SS}} \frac{\sum_{i=1}^{n} v_{x(i)}(x(i))}{\sum_{i=1}^{n} v_i(x(i))}$$

where $\mathcal{SS}$ here denotes the space of swap-stable assignments

With all the relevant definitions, we now present the first welfare guarantee for random roscas with after-markets

Lemma 3.4. *Any assignment that is a swap stable has a price of anarchy at most 2*

Proof. For all, i, j swap-stable implies that

$$v_i(x(i)) + v_j(x(j)) \geq v_i(x(j)) + v_j(x(i)) \tag{3}$$

Pick a specific i and let $j = x(i)$ equation 3 becomes

$$v_i(x(i)) + v_{x(i)}(x(x(i))) \geq v_i(x(x(i))) + v_{x(i)}(x(i)) \tag{4}$$

Summing across all i, since every person is assigned a pot (4) becomes

$$2\sum_i v_i(x(i)) \geq \sum_i v_{x(i)}(x(i)) + \sum_i v_i(x(x(i)))$$

Since $v_i(\cdot) \geq 0$

$$2\sum_i v_i(x(i)) \geq \sum_i v_{x(i)}(x(i))$$

But note the term on the right is just the welfare of the optimal assignment thus.

$$2\sum_i v_i(x(i)) \geq \text{OPT}(v)$$

$$2 \geq \frac{\text{OPT}(v)}{\sum_i v_i(x(i))} = \frac{\text{OPT}(v)}{\textsc{welf}(v, x)}$$

and thus the price of anarchy of any swap-stable assignment x is at most 2. □

The following example constructed using linear programming shows that this bound is tight for swap-stable matchings.

Example 3.5. Consider a 3-person Rosca let $v_1 = \{(1, 1), (2, 0), (3, 0)\}$, $v_2 = \{(1, 1), (2, 1), (3, 0)\}$, $v_3 = \{(1, 0), (2, 0), (3, 0)\}$ Then $x = \{(1, 3), (2, 1), (3, 2)\}$ is a swap-stable assignment however the welfare achieved by the optimal assignment $v_1(1) + v_2(2) + v_3(3) = 2$ while the welfare achieved by our assignment $v_1(3) + v_2(1) + v_3(2) = 1$. Thus swap-stable assignments have a price of anarchy exactly 2.

Note that in this example people's values are nonincreasing across time, this is interesting as we will see in later sections this property is often helpful in improving the price of anarchy in roscas[1]. For full information and code used to construct this example see Appendix B. Its easy to see that just the lottery part of the RRAM is no better than an n-approximation: just give everyone value 0, except for one person, who needs the money in round 1. So the after-market is a powerful mechanism for achieving high welfare in as shown by the fact that as the number of participants grows large the welfare contributions from the initial assignment are meaningless

[1]In the case of RRAMs, this property doesn't improve POA as the example also proves our bound is tight for nonincreasing values

4 TECHNICAL BACKGROUND

4.1 Analytical Challenges for Bidding Roscas

Roscas which use some kind of auction to determine the receipt order are the most prevalent after random roscas/RRAM see Ardener [6]. People's bid in the rosca auctions are in terms of contributions to the pot - either higher contributions or discounts to the other members. Typically these are held using *oral ascending or descending bid auctions* in each round. These and the other main types of single-item auctions that are relevant to this paper are described below.

Definition 4.1 (Ascending-bid auctions). Ascending-bid auctions, also called English auctions. These auctions are carried out usually in real time, with bidders present either physically or virtually. The seller gradually raises the price, bidders drop out until only one bidder remains, and that bidder wins the item at this final price.

Definition 4.2 (Descending-bid auctions). Descending-bid auctions, also called Dutch auctions. This is also an interactive auction format, in which the seller gradually lowers the price from some high starting value until a bidder accepts and pays the current price.

Definition 4.3 (First-price sealed-bid auctions). First-price sealed-bid auctions(FPA). In this kind of auction, bidders submit simultaneous "sealed bids" to the seller. The terminology comes from the original format for such auctions, in which bids were written down and provided in sealed envelopes to the seller, who would then open them all together. The highest bidder wins the object and pays the value of their bid to the seller.

Definition 4.4 (Second-price sealed-bid auctions). Second-price sealed-bid auctions (SPA) , also called Vickrey auctions. Bidders submit simultaneous sealed bids to the sellers; the highest bidder wins the object and pays the value of the second-highest bid to the seller.

The ascending and descending-bid auctions have equivalent "outcomes" to the second and first price auctions respectively. For the ascending-bid and second-price auctions you can see why from thinking about an ascending-bid auction, in which bidders gradually drop out as the seller steadily raises the price. The winner of the auction is the last bidder remaining, and they pay the price at which the second-to-last bidder drops out. For an argument for the descending-bid and first-price auction see Chapter 9 of [25]. For these reasons we will restrict our analysis to the FPA and SPA.

Theoretical studies of the auction outcomes in roscas have proved challenging for three key reasons. First, roscas are dynamic: participants make decisions in an online fashion, and condition their future decisions on past outcomes. Second, as participants are often uncertain about each others' financial needs, a theory of roscas must accommodate incomplete information. Finally, the sums of money in play in roscas are significant enough that people's utilities are nonlinear. The standard economic approach of solving for auction equilibrium under any one of these phenomena in isolation is already challenging. As highlighted in Section 2 more comprehensive approaches at rosca analysis have only succeeded with aggressive simplifying assumptions. For these reasons theory developed and adapted to analyze bidding roscas is significantly more involved than that of RRAM and so in the following section we will take some time motivating and providing the necessary background to understand the analyses for bidding roscas.

4.2 Algorithmic Mechanism Design

In order to model and analyze welfare in roscas, this work primarily uses tools developed in the algorithmic game theory (AGT) literature. Here, we introduce the necessary concepts and terminology from the AGT literature; for a comprehensive and more general definitions, see Nisan et al. [40]. First, we define basic terminology then we discuss how we capture the idea of a stable/final outcome in a strategic situation.

A *game* or *mechanism* is any interaction between intelligent and strategic people (usually referred to as *agents*) where agents' *actions* or *strategies* affect their's and other's outcomes. A large number of everyday interactions can be modeled as games and thus thinking formally about and developing the theory of games is broadly important. For example, the Oberlin Computer Science Department (OCCS) wants to give students a chance to explore a topic they are interested in at a high level through the honors program. However, there is a limited number of faculty with a limited capacity to help students with an honors project so they could come up with the *"honors application game"* to help them effectively match professors to students. Factoring in what kinds of outcomes they want, they decide the *rules* of the application process/game. For the n students/agents, $\{1, 2, \ldots, n\}$ who are interested in doing honors with some professor, the rules of the game determine the *set of possible actions/strategies* S_i for each agent i. To participate in the application game, each agent i selects a strategy $s_i \in S_i$, e.g which professor to target in their application. We will use $s = (s_1, \ldots, s_n) = (s_i, s_{-i})$ to denote the *vector of strategies* selected by the agents, s_i is the strategy agent i picks, s_{-i} is the $(n-1)$-dimensional vector of the strategies played by all other agents and $S = \times_i S_i$ denotes the set of all possible ways agents could pick strategies.

The actions $s \in S$ selected by the agents determines the outcome for each agent, and agents would prefer some outcomes over others. We specify agents' preferences over outcomes using the numerical values given by the utility function $u_i : S \to \mathbb{R}$. Thus $u_i(s)$ – or equivalently $u_i(s_i, s_{-i})$ – denotes an agent i's utility from a certain outcome. In our illustrative example, since students are trying to maximize their chances of getting matched with a professor, they factor in all the information available, consider their resources and try to think about how the other students and the professors are going to act. Since the students are rational they act in a way that maximizes their chances of achieving a desirable outcome. If every agents is acting that way, then their actions form a stable outcome or *equilibrium*.

Definition 4.5 (Nash Equilibrium). A strategy vector $s \in S$ is said to be a *Nash Equilibrium* if for all players i and each alternate strategy $s_i' \in S_i$

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \tag{5}$$

This corresponds to the concept of a stable outcome because it says no one has any utility to gain from unilaterally deviating from the current strategy. As you might expect such a strategy might not always exist. In addition, a student might not know how many other students are applying or what their preferences are we say they have *incomplete information*. What strategy maximizes payoff depends on what the other students' preferences are, so students must now pick their strategies from a distribution based on their beliefs about other students' preferences. Such a randomized strategy is called a *mixed strategy* and always exist for any game with a finite set of players and strategies [40], though they are generally hard to find (see Daskalakis et al. [23]). Students have some idea of what other students' preferences might be based on previous years and other experiences, thus they have a distribution on the preferences of the other students and can now pick a strategy to maximize their expected payoff.

Definition 4.6 (Bayes-Nash Equilibrium). A strategy vector $s \in S$ is said to be a *Bayes-Nash Equilibrium* (BNE) if for all players i and each alternate strategy $s_i' \in S_i$

$$\mathbb{E}[u_i(s_i, s_{-i})] \geq \mathbb{E}[u_i(s_i', s_{-i})] \tag{6}$$

Bayes-Nash Equilibria allow you to predict how rational people should/would act in a game. *Mechanism design* aims to design games that have equilibria with desirable outcomes to the mechanism designer. Henceforth we will use s to denote an equilibrium strategy while a represents a more general action or strategy that might not necessarily be an equilibrium.

5 FORMAL MODEL OF ROSCAS

We now present our model of Bidding Roscas, following [13], [14], as in Section 3 a rosca for n agents takes place over n discrete time periods. Each period, three things occur. (1) Each agent pays an amount p_0 into a common pot. (2) A winner for the np_0 units of money in the pot is decided among those who have not yet won. (3) In roscas that allocate by auction, we assume payments are distributed evenly amongst the non-winning members as a third step in the process, as is common in practice. We refer to this latter process as the *internalized seller*.

Roscas give individuals without access to banking a way to save for substantial investments such as large durable goods. We assume each agent desires to pay for a one-time investment, yielding for agent i a utility stream of ξ_i^t units at period t if and only if the agent has invested in the past. We follow [13] and assume the cost of the investment is homogeneous across agents, and equal to the pot amount np_0. However, we allow ξ_i^t to vary across time and across agents. We further assume each agent has concave, nonnegative utility U for wealth consumption each period. Concave utillities is the standard way to model risk-averse agents (like the typical rosca participant) as small changes in wealth would lead to large changes in utility when an agent has low wealth. Each period t, each agent has initial wealth level w. After paying p_0 into the pot and making additional payments p_i^t to the auction mechanism, the agent consumes their remaining wealth for period t for utility $U(w - p_0 - p_i^t)$. Let $\mathbb{I}_i^t$ indicate whether the agent has invested by time t. An agent's utility at time t is thus $U_i(w_i - p_0 - p_i^t) + \xi_i^t \mathbb{I}_i^t$.

We study the utilitarian social welfare of the rosca, given by

$$\text{WELF} = \sum\nolimits_i \sum\nolimits_t \xi_i^t \mathbb{I}_i^t + \sum\nolimits_i \sum\nolimits_t U(w - p_0 - p_i^t). \tag{7}$$

As our benchmark, we consider the optimal ordering for pot allocation, i.e. one-to-one function j^* from agents to time periods maximizing the sum of agents' utility streams. Note that we may also allow wealth redistribution, but the concavity of U implies any such redistribution is suboptimal. Hence, our welfare benchmark is given by

$$\text{OPT} = \max_{j*:[n]\to[n]} \sum_i \sum_{t=j*(i)}^{n} \xi_i^t + n^2 U(w - p_0),$$

The summation in OPT is the utility agents get from allocation OPT this is maximized in the optimal allocation. The second term is the base utility agents enjoy from their starting wealth after they've made their rosca contributions. The n^2 comes from the fact that n agents are making payments in n periods. We seek to measure the worst-case approximation ratio between WELF and OPT i.e the price of anarchy.

We finally note three assumptions on U. First, we have assumed homogeneity of U. This is valid in practice: participants in roscas tend to possess similar socioeconomic status (see [38], [19]), and thus it is reasonable to assume similar utility for wealth [2]. Second, we will assume in what follows that the slope of U is bounded. This matches the observation that investments from roscas tend to be significant (hence $U'(x) := 1$ is unreasonable), but not so large as to dwarf individuals' livelihoods [8]. We will parametrize our welfare approximations by the bounds on U's slope. Finally, note that

[2]Note that similar socioeconomic status doesn't preclude varying investment opportunities or needs for large outlays of cash, and hence the ξ_i^t may yet be heterogeneous.

multiplicative approximation guarantees only improve as $U(w - p_0)$ grows. It will therefore be entirely without loss to take $U(w - p_0) = 0$.

5.1 Roscas as Auctions

We now give definitions necessary to study the bidding rosca. In the process, we will recast roscas in more standard auction-theoretic notation.

A *multi-round allocation setting* consists of n agents and m items to be allocated, one item per period An allocation consists of a mapping from the items to the agents. Each agent has a real-valued valuation $v_i = (v_i^1, \ldots, v_i^m)$ over the items [3], and agents are *unit-demand*: their value [4] for a set $\mathfrak{S}$ of items is $\max_{j \in \mathfrak{S}} v_i^j$. Let $\mathcal{V}_i$ be the set of possible valuations of agent i. An outcome in a multi-round mechanism is an allocation x and payment vector p. Allocations are given by $x = (x_1, \ldots, x_n)$, where each $x_i = (x_i^1, \ldots, x_i^n)$ is an indicator vector, and $x_i^j = 1$ if and only if i receives item j. Denote by $\mathcal{X}$ the space of feasible allocations. Payments are of the form $p_i = (p_i^1, \ldots, p_i^m)$, with each agent making payments each period. We assume agents' utilities to be additively-separable with convex disutility function C (defined below) for payments:

$$u_i^{v_i}(x_i, p_i) = v_i \cdot x_i - \sum_j C(p_i^j).$$

The rosca setting maps into this framework in the following way: each pot in a rosca is a distinct item (with $m = n$). An agent i with utilities $\xi_i^1, \ldots, \xi_i^n$ then has value $v_i^j = \sum_{k=j}^n \xi_i^k$ for winning the jth pot (and with later pots providing no additional value). Note that in this formulation, v_i^j is decreasing in j since agents only gain utility after they win a pot, and increasing j implies winning the pot later in time. The disutility function C for payments in a rosca can be taken to be $C(p_i^j) = -U(w - p_0 - p_i^j)$, since $\sum_j \xi_i^j \mathbb{I}_i^j + \sum_j U(w - p_0 - p_i^j) = v_i \cdot x_i - \sum_j C(p_i^j)$ and $\sum_j \xi_i^j \mathbb{I}_i^j = v_i \cdot x_i$ by definition. Under these assumptions, we have $\sum_i u_i^{v_i}(x_i, p_i)$ is exactly equal to the social welfare given by (7).

We assume C is non-decreasing in p which is equivalent to U being decreasing with payments and $C(0) = 0$, which is equivalent to $U(w - p_0) = 0$. Since U's slope is bounded this is equivalent to saying that $\alpha \leq C'(p) \leq \beta$ where $\alpha, \beta > 0$. These assumptions imply some convenient properties of $C(\cdot)$:

i. $\alpha x \leq C(x) \leq \beta x$, $-\alpha x \leq C(x)$, $-C(x) \leq \beta x$ for any $x \geq 0$
ii. $-\alpha x \leq -C(x) \leq -\beta x$, $\alpha x \leq -C(x)$, $C(x) \leq -\beta x$ for any $x \leq 0$

Some of our results hold for general mechanism not just allocation settings so it is useful to define more general notation as well. However, picture an allocation setting like an auction when trying to think of concrete mechanisms. It is also useful to differentiate between equilibrium and non-equilibrium strategies, henceforth we will adopt the convention that s denotes an equilibrium strategy while a represents a more general action.

In round $j \in [m]$ of a multi-round mechanism, the mechanism takes a profile of actions $a^j = (a_1^j, \ldots, a_n^j)$ and outputs an allocation $X^j(a^j)$ of item j and a profile of payments $P^j(a^j)$. The mechanism may condition X^j and P^j on previous rounds' actions. Typical pot allocation procedures in roscas resemble standard single-item auctions, with the additional restrictions that agents who won in previous rounds are ineligible for allocation, and that payments are redistributed among all agents.

[3]We refer to items in our abstraction to multi-item auctions and pots in the rosca application interchangeably.

[4]We will often alternate between writing valuations/allocation as vectors v_i^j, x_i^j or as vector valued functions $v_i(x_i^j), X_i(a)$ for notation simplicity. Note, the output of the valuation/allocation functions could be 1x1 vectors. It should be clear from context the dimensions of the output.

We will consider mechanisms that are *individually rational* in each round i.e each agents utility is nonnegative at the end of the mechanism. Thus, for any profile of actions in round j, the only agent with positive payments can be the winner of item j. We will further assume that the winner's payments are redistributed among the losers. For example, in round j of a rosca with a first-price rule, agents submit bids $b_1^j, \ldots b_n^j$. Among those who have not yet won the pot, the highest bidder i^* wins, and payments are $P_{i^*}^j = b_{i^*}^j$ and $P_i^j = b_{i^*}^j/(n-1)$ for $i \neq i^*$. We will show that not just first- and second-price auctions, but any "reasonble" (i.e. smooth, defined in Section 6) single-item auctions can form the basis for an approximately-optimal rosca.

We study dynamic equilibria of roscas under incomplete information on agents' values for their investment opportunities. In auction notation, each agent draws their profile of values $v_i = (v_i^1, \ldots, v_i^n)$ from a distribution F_i, which is independent across agents, but correlated across rounds (in particular, decreasing across rounds). Agents can observe histories of play in past rounds, and hence a strategy s_i is a mapping s_i^j for each round j from values and histories of past play to actions/bids in round j. A profile of strategies $s = (s_1, \ldots, s_n)$ is a *Bayes-Nash equilibrium* if each agent's strategy maximizes their total utility across rounds, taken in expectation over other agents' values. In what follows, we fix an auction format for pot distribution, and bound the worst-case welfare approximation of roscas in the worst-case Bayes-Nash equilibrium, taken over all value distributions and equilibrium strategy profiles for those value distributions. That is, we study:

$$\max_{F,\, s \in \mathrm{BNE}(F)} \frac{\mathbb{E}_{v \sim F}[\mathrm{OPT}(v)]}{\mathbb{E}_{v \sim F}[\sum_i u_i^{v_i}(X_i(s(v)), P_i(s(v)))]},$$

i.e the *price of anarchy* of the mechanism where $\mathrm{OPT}(v) = \max_{x \in \mathcal{X}} \sum_i v_i \cdot x_i$.

6 SMOOTHNESS FOR ROSCAS

To introduce our analysis technique of smoothness, we briefly go over relevant definitions and theorems of smoothness from Syrgkanis and Tardos [46]. Then we show how the theory of smoothness can be adapted for Roscas.

Definition 6.1 (Smoothness). A mechanism is (λ, μ)-smooth $\lambda, \mu \geq 0$ if for any valuation profile $v \in \times_i \mathcal{V}_i$ and for any action profile a there exists a randomized action $\mathbf{a}_i^*(v, a_i)$ for each player i such that the following holds:

$$\sum_i u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \lambda \mathrm{OPT}(v) - \mu \sum_i P_i(a). \tag{8}$$

Smoothness trades off revenue against an agents utility for deviating from their current actions. The definition of a smooth mechanism has a very natural interpretation as guaranteeing an approximate analog of market cleaning prices i.e utilities are maximized, all buyers have been serviced and all inventory has been sold. Bikhchandani [15] showed that pure Nash equilibria of a simultaneous first price auction have market clearing prices, and this implies that the outcome is efficient. Aggregate market clearing prices are guaranteed when each agent can modify their bid to claim their optimal allocation at the price paid for this allocation in the current solution. $(1, 1)$-smoothness in essence requires this property only in aggregate, but for any outcome of the mechanism, not only at equilibrium. While (λ, μ)-smoothness requires this only approximately, both in terms of the allocation claimed, as well as the price paid for it. In addition, unlike the pure equilibrium analysis, it requires the modified bid to be ignorant of the actions of the rest of the players.

Lemma 6.2. *If a mechanism is (λ, μ)-smooth and individually rational then the price of anarchy of the mechanism is at most $\lambda/\max(\mu, 1)$*

Proof. See Theorem 4.2 in Syrgkanis and Tardos [46] □

Example 6.3 (Second-Price Auction not Smooth). Take a 2-person SPA with quasilinear utilities (i.e $C(x) = x$). If $v_1 = 1, v_2 = 0$ and if $b_1 = 0$ and $b_2 = 1 + \epsilon$ where $\epsilon > 0$. Then this constitutes an equilibrium as agent 1 wouldn't want to change their bid as they would have to bid higher than $1 + \epsilon$ to change the outcome but this results in $-\epsilon < 0$ utility. Agent 2 wouldnt want to change their bid because the maximum utility they can ever get is 0 which is what they are getting in this outcome. However, this example violates the smoothness condition and its price of anarchy is unbounded.

Such degenerate examples can be avoided by imposing a no overbidding assumption. Under such an assumption a weaker notion of smoothness is defined in [46] to capture mechanisms that produce high efficiency under a no-overbidding refinement. The second-price auction satisfies this weaker notion of smoothness and this is the weak smoothness we generalize in (Definition 6.5).

6.1 Smoothness with Internalized Sellers

The standard smoothness approach trades off revenue against agents' deviation utilities. In auctions with an internalized auctioneer, the former quantity is always zero. We now give an adapted definition that generalizes to multi-round allocation mechanisms (the standard definition comes when the number of rounds is 1) and accounts for the rebates made each round. For mechanisms which are *individually rational* each round, the winners are the only agents generating payments to be redistributed.

Definition 6.4 (Willingness-to-pay before Rebates). Let $R_i(a)$ denote the rebates agent i enjoys under action profile a i.e the money generated by the mechanism that is redistributed to the losing agents. Then the *maximum willingness-to-pay* before rebates for an allocation x_i when using strategy a_i is defined as the maximum they could ever pay before rebates conditional on allocation x_i

$$B_i(a_i, x_i) = \max_{a_{-i}:X_i(a)=x_i} C(P_i(a) + R_i(a))$$

Note, without internalized sellers (rebates = 0) and with quasilinear bidders ($C(x) = x$) this definition is exactly the definition of willingness-to-pay from [46].

Definition 6.5 (Weakly Smooth before Rebates). A multi-round allocation mechanism is weakly $(\lambda, \mu_1, \mu_2, \mu_3)$*-smooth before rebates* $\lambda, \mu_1, \mu_2, \mu_3 \geq 0$ if for any valuation profile $\times_i \mathcal{V}_i$ and for any action profile a there exists a randomized action $\mathbf{a}_i^*(v, a_i)$ for each player i such that the following holds:

$$\sum_i u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \lambda \mathrm{OPT}(v) - \mu_1 \sum_i \sum_j P_i^j(a) - \mu_2 \sum_i \sum_j R_i^j(a) - \mu_3 \sum_i \sum_j B_i^j(a, X(a)) \quad (9)$$

Definition 6.6 (Smooth before Rebates). A multi-round allocation mechanism is (λ, μ_1, μ_2)*-smooth before rebates* if its weakly $(\lambda, \mu_1, \mu_2, 0)$-smooth before rebates

Definition 6.7 (No Overbidding). A strategy profile a satisfies round-wise *no overbidding* if

$$\mathbb{E}_a[B_i^j(a_i, X_i(a))] \leq \mathbb{E}_a[v_i^j(X_i^j(a))]$$

Note this is the same as saying $P_i^j(a) + R_i^j(a) \leq C^{-1}(v_i^j(X_i^j(a)))$

Claim 1. *All the welfare guarantees in this paper hold for smooth mechanisms (strong form) without a no overbidding assumption.*

Proof Sketch. First, note mathematically that in all our coming proofs we only use the no overbidding assumption to bound $B_i^j(a_i, X_i(a))$ and the coefficient μ_3 on this term is 0 in a smooth mechanisms. Unnatural equilibria where agents overpay in an early round to induce higher payments (and therefore rebates) from their competition later on cannot hurts us. Why? Rebates in each round have to be less than $C^{-1}(v_{w_j}(x_{w_j}))$ where w_j denotes the agent who won the item in round j. Since our mechanisms are individually rational w_j is the only one making positive payments each round and thus they have to pay less than $C^{-1}(v_{w_j}(x_{w_j}))$ to avoid negative utility. Thus if an agents is overbidding to get higher rebates in future rounds this only works if their overbid causes them to win in strongly smooth mechanisms but as we showed earlier the winning agent in each round is never overbidding. □

Lemma 6.8. *For any multi-round allocation mechanism that is individually rational each round and is weakly (λ, μ_1, μ_2)-smooth, the same mechanism with an internalized seller is weakly $(\lambda, \mu_1, \mu_1, \mu_2)$-smooth before rebates.*

Proof. Let M be a mechanism satisfying the conditions of the lemma, and $\hat{M}$ the version with an internalized seller. Denote by $u_i^{v_i}$, P_i^j, R_i^j, X_i, and B_i^j the utilities, payments, rebates, allocations, and willingness to pay in M, and let $\hat{u}_i^{v_i}$, $\hat{P}_i^j$, $\hat{R}_i^j$, $\hat{X}_i$, and $\hat{B}_i^j$ denote the same quantities in M with an internalized seller. For all action profiles a, $\hat{u}_i^{v_i}(a) \geq u_i^{v_i}(a)$. By individual rationality and the fact that $\hat{M}$ redistributes to losers each round, we have $\sum_i P_i^j(a) = \sum_i \hat{P}_i^j(a) + \hat{R}_i^j(a)$ for all j. Since $\sum_i R_i^j(a) = 0$ as there are no rebates in the original mechanism, $\sum_i P_i^j(a) + R_i^j(a) = \sum_i \hat{P}_i^j(a) + \hat{R}_i^j(a)$.Although the presence of internalized sellers doesn't affect payments into the mechanism, it does affect the net payments by reducing your payments through rebates; however, by adding back the rebates when calculating an agents willingness to pay we get that $B_i^j(a) = \hat{B}_i^j(a)$ □

6.2 Bayesian Extension

A mechanism that is smooth before rebates trades off revenue and rebates against bidder utilities for any fixed pair of value and action profiles. This extends to a Bayesian tradeoff as well. We assume the mechanism gives agents the option to withdraw (e.g. by bidding 0) and receive 0 utility.

Lemma 6.9. *If an individually rational mechanism $\mathcal{M}$ with an internalized seller is weakly $(\lambda, \mu_1, \mu_2, \mu_3)$-smooth before rebates and agents have the option to withdraw, then if buyers have disutility for payments satisfying $C(0) = 0$ and $C'(x) \in [\alpha, \beta]$ for all x, value distributions are independent across agents, and values are nonincreasing, then the price of anarchy in Bayes-Nash equilibrium of M satisfying no overbidding is at most $(\alpha + \max(\mu_1, \mu_2) + (\alpha + \beta)\mu_3)/\lambda\alpha$*

Proof. Let s be a Bayes-Nash equilibrium. For a bidder i, consider the following "bluff" deviation [5] which lowerbounds their expected equilibrium utility: have them sample a value profile w according to agents' distributions, and play the deviation $a_i^*((v_i, w_{-i}), s_i(w_i))$ given by smoothness before rebates. Agent i's expected utility from this deviation is at least:

$$\mathop{\mathbb{E}}_{v,w}[u_i^{v_i}(a_i^*((v_i, w_{-i}), s_i(w_i)), s_{-i}(v_{-i}))] = \mathop{\mathbb{E}}_{v,w}[u_i^{w_i}(a_i^*((w), s_i(v_i)), s_{-i}(v_{-i}))],$$

[5]This "bluffing" technique was first used in the proof of Theorem 4.3 in [46] for a similar result.

where the latter equality follows from exchanging the roles of v and w. Summing over agents:

$$\mathop{\mathbb{E}}_{v,w}\Big[\sum\nolimits_i u_i^{w_i}(a_i^*((w),s_i(v_i)),s_{-i}(v_{-i}))\Big] \geq \lambda \mathop{\mathbb{E}}_{v,w}[\mathrm{OPT}(w)] - \mu_1 \mathop{\mathbb{E}}_{v,w}\left[\sum_i\sum_j P_i^j(s(v))\right]$$
$$- \mu_2 \mathop{\mathbb{E}}_{v,w}\left[\sum_i\sum_j R_i^j(s(v))\right] - \mu_3 \mathop{\mathbb{E}}_{v,w}\left[\sum_i\sum_j B_i^j(s(v),X(s(v)))\right]$$

Note that on the right side of the inequality each expectation only depends on w or v, but not both. For the left side, since agents are best responding in equilibrium, we obtain

$$\mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big] \geq \lambda \mathop{\mathbb{E}}_{v}[\mathrm{OPT}(v)] - \mu_1 \mathop{\mathbb{E}}_{v}[\sum_i\sum_j P_i^j(s(v))] - \mu_2 \mathop{\mathbb{E}}_{v}[\sum_i\sum_j R_i^j(s(v))] \tag{10}$$
$$- \mu_3 \mathop{\mathbb{E}}_{v}[\sum_i\sum_j B_i^j(s(v),X(s(v)))].$$

The final task is to convert the above inequality into a welfare guarantee. We may write agents' utilities as

$$\mathop{\mathbb{E}}_{v}[u_i^{v_i}(s(v))] = \mathop{\mathbb{E}}_{v}\Big[v_i \cdot X_i(s(v)) - \sum\nolimits_j C(P_i^j(s(v))).\Big]$$

With an internalized seller, we have

$$\sum_i\sum_j (R_i^j(s(v)) - P_i^j(s(v))) = \sum_i\sum_j (P_i^j(s(v)) + R_i^j(s(v)))$$

Since $\alpha(R_i^j(s(v)) - P_i^j(s(v))) \leq -C(R_i^j(s(v)) - P_i^j(s(v)))$ since $(R_i^j(s(v)) - P_i^j(s(v))) \leq 0$ and individual rationality implies $u_i(s(v)) \geq 0$ then we obtain that

$$\mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j (P_i^j(s(v)) + R_i^j(s(v)))\Big] \leq \alpha^{-1} \mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j -C(R_i^j(s(v)) - P_i^j(s(v)))\Big] \quad \leq \alpha^{-1} \mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big].$$

thus

$$\sum_i\sum_j P_i^j(s(v)) + \sum_i\sum_j R_i^j(s(v)) \leq \alpha^{-1} \mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big]. \tag{11}$$

Now we bound the welfare contributions from willingness-to-pay. Since no overbidding implies that

$$\mathop{\mathbb{E}}_{v}\Big[\sum_j B_i^j(s(v),X_i(s(v)))\Big] \leq \mathbb{E}_v[v_i \cdot X_i(s(v))]$$

we get that

$$\mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big] = \mathop{\mathbb{E}}_{v}\Big[\sum_i v_i \cdot X_i(s(v)) - \sum_j C(P_i^j(s(v))).\Big]$$
$$\geq \mathop{\mathbb{E}}_{v}\Big[\sum_i \sum\nolimits_j B_i^j(s(v),X_i(s(v))) - \sum\nolimits_j C(P_i^j(s(v))).\Big]$$
$$\mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big] \geq \mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j B_i^j(s(v),X_i(s(v)))\Big] - \mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j C(P_i^j(s(v)) + R_i^j(s(v)))\Big]$$
$$\mathop{\mathbb{E}}_{v}\Big[\sum_i u_i^{v_i}(s(v))\Big] \geq \mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j B_i^j(s(v),X_i(s(v)))\Big] - \beta \mathop{\mathbb{E}}_{v}\Big[\sum_i\sum_j P_i^j(s(v)) + R_i^j(s(v))\Big]$$

By the inequality (11)

$$\mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big] \geq \mathop{\mathbb{E}}_{v}\Big[\sum_{i}\sum_{j} B_i^j(s(v), X_i(s(v)))\Big] - \beta/\alpha \mathop{\mathbb{E}}_{v}\Big[u_i^{v_i}(s(v))\Big]$$

$$(1+\beta/\alpha)\mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big] \geq \mathop{\mathbb{E}}_{v}\Big[\sum_{i}\sum_{j} B_i^j(a_i, X_i(a))\Big]$$

Note that $\max(\mu_1,\mu_2)P_i^j(s(v)) + max(\mu_1,\mu_2)R_i^j(s(v)) \geq \mu_1 P_i^j(s(v)) + \mu_2 R_i^j(s(v))$. So equation (10) becomes

$$\mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big] \geq \lambda \mathop{\mathbb{E}}_{v}[\mathrm{OPT}(v)] - \max(\mu_1,\mu_2)/\alpha \mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big] - \mu_3(1+\beta/\alpha)\mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big]$$

$$(\alpha + \max(\mu_1,\mu_2) + (\alpha+\beta)\mu_3)/\alpha \mathop{\mathbb{E}}_{v}\Big[\sum_{i} u_i^{v_i}(s(v))\Big] \geq \lambda \mathop{\mathbb{E}}_{v}[\mathrm{OPT}(v)]$$

Thus,

$$(\alpha + \max(\mu_1,\mu_2) + (\alpha+\beta)\mu_3)/\lambda\alpha \geq \mathop{\mathbb{E}}_{v}\left[\frac{\mathrm{OPT}(v)}{\sum_i u_i^{v_i}(s(v))}\right]$$

□

7 ROUND-ROBIN COMPOSITION

We now reduce the problem of designing a smooth rosca to the much simpler task of designing smooth single-item auctions. In the standard formulation of Syrgkanis and Tardos [46], smooth mechanisms are closed under sequential composition with unit-demand agents. In other words, if m items are sold sequentially, and each item is sold with a mechanism which is smooth in isolation, then the composite mechanism will also be smooth. Roscas have a similar structure, with the key difference that agents who have won in earlier rounds are ineligible in later rounds. This difference imposes a surprising obstacle, and rules out a generalization of Syrgkanis and Tardos [46]. However under certain assumptions on agents preferences across time we are able to obtain an extension theorem.

We first define composition for roscas. A rosca's pot allocation procedure for a given round is typically a single-item auction, given by M, with action space $\mathcal{A} = \times\mathcal{A}_i$, allocation rule X, and payment rule P. We assume M has a "withdraw" action $\perp$ which guarantees agents nonpositive payments. We allow $\perp$ to induce negative payments, as we allow mechanisms with an internalized seller, where winners' payments are redistributed to losers. A rosca can then be thought of as n copies of M composed in the following sense.

Definition 7.1 (Round-Robin Composition). Given single-item auction $M = (\mathcal{A}, X, P)$ with n agents, the *n-item round-robin composition* of M is a multi-round allocation mechanism for n items using the following procedure. Each round j, agents submit actions $a^j = (a_1^j, \ldots, a_n^j)$. The mechanism sets $\underline{a}_i^j = a_i^j$ for any i who have not yet won an item, and $\underline{a}_i^j = \perp$ for all i who have won an item previously. The mechanism then allocates item j to agents according to M applied to $\underline{a}^j$ and assigns payments for round j accordingly.

We assume agents can condition their round j actions on play and outcomes in rounds $1, \ldots, j-1$. A multi-round allocation mechanism that is the round-robin composition of a smooth mechanism need not be smooth itself, due to two factors. First, agents can condition their actions on past play, and hence a small change in an agent's early bids might induce arbitrary behavior later. Second, there are action profiles where agents might have high value for later items, but win in early rounds. Smoothness deviations in these circumstances may not exist, as the following example demonstrates.

Example 7.2. Consider a two-round, two-agent rosca with pots allocated by first-price auction. Values will be $v_1 = (0, 1)$ and $v_2 = (0, 0)$. Actions are contingency plans over bids. Our action profile a will be he following: agent 1 bids ϵ in round 1 and 0 in round 2. Agent 2 bids 0 in round 1, and then conditions their round 2 bid on agent 1's round 1 behavior. If agent 1 bids ϵ in round 1, agent 2 bids 0 in round 2. Otherwise, agent 2 bids an arbitrarily large number Ω. Agent 1 does not have a smoothness deviation that earns them any utility, and the revenue from a is negligible. Hence, smoothness is not preserved under round-robin composition with general values.

Definition 7.3 (Nonincreasing Values). For a multi-round allocation setting, let x_i^j denote the allocation where agent i wins in round j and loses in all other rounds. A value function v_i in a multi-round allocation setting is *nonincreasing with time* if $v_i(x_i^j) \leq v_i(x_i^{j'})$ for all $j > j'$.

Non-increasing values capture the idea that all things equal agents would rather receive the pot earlier than later.In roscas, an agent i's value v_i^j for winning the pot in round j is their total income stream in subsequent rounds, $\sum_{k=j}^{n} \xi_i^k$. Consequently, v_i^j is nonincreasing in j. Nonincreasing values allows us to prove the round robin composition property of smoothness, with loss in the smoothness parameters. We obtain:

Theorem 7.4. *Let M be a weakly $(\lambda, \mu_1, \mu_2, \mu_3)$-smooth before rebates, individually rational single-item mechanism. Then if agents' values are nonincreasing with time, then the round-robin composition is weakly $(\min(1, \lambda), 1 + \mu_1, 1 + \mu_2, \mu_3)$-smooth before rebates.*

The proof uses the smoothness deviations for the single-item mechanism to explicitly construct deviations for the composition, using a similar strategy to the proof of Lemma A.1.

Proof. For an n-item setting, an optimal allocation given value profile v is an assignment from agents i to items j_i^*. For values v and action profile a, we construct a deviation $\mathbf{a}_i^*(v, a_i)$ for each agent i in the following way. Agent i simulates their equilibrium strategy up until round $j_i^* - 1$ and then in j_i^* they play their smoothness deviation for M on value profile $\hat{v}$ and action a_i^j for i, where $\hat{v}_i = v_i^j$ and 0 for all other agents. They withdraw in all subsequent rounds. If, in simulating a_i, agent i wins in some round $\hat{j}_i \leq j_i^*$ they similarly withdraw in all subsequent rounds, and do not play their smoothness deviation at j_i^*.

Let S_{EQ} denote the set of agents whose deviations cause them to win before j_i^* and let S_{OPT} denote the set of agents who don't win before j_i^* hence are able to play their smoothness deviation in round j_i^*. For agents in S_{OPT}, the choice of smoothness deviation for round j_i^* implies:

$$u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \lambda \cdot v_i^{j_i^*} - \mu_1 \sum_i P_i^{j_i^*}(a) - \mu_2 \sum_i R_i^{j_i^*}(a) - \mu_3 \sum_i B_k^{j_i^*}(a_i, X_i(a)),$$

For agents in S_{EQ} if $\hat{j}_i$ denotes the round they actually win in then

$$u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) + P_i^{\hat{j}_i}(a) + R_i^{\hat{j}_i}(a) \geq v_i(x_i^{\hat{j}_i}) \geq v_i(x_i^{j_i^*}),$$

where the first inequality follows from the fact that i's utility and payment in round $\hat{j}_i$ sum to $v_i(x_i^{\hat{j}_i})$ and they get 0 rebates as a winner, and the second from the nonincreasing values property. Summing over agents, we obtain:

$$\sum_{i \in S_{\text{OPT}}} \left[u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) + \mu_1 \sum_k P_k^{j_i^*}(a) + \mu_2 \sum_k R_k^{j_i^*}(a) + \mu_3 \sum_k B_k^{j_i^*}(a_i, X_i(a))) \right]$$
$$+ \sum_{i \in S_{\text{EQ}}} \left[u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) + P_i^{\hat{j}_i}(a) + R_i^{\hat{j}_i}(a) \right] \geq \min(1, \lambda)\text{OPT}(v).$$

Furthermore, we have that both of:

$$\sum_{i \in S_{\mathrm{OPT}}} \sum_k P_k^{j_i^*}(a) \quad \text{and} \quad \sum_{i \in S_{\mathrm{EQ}}} P_k^{\hat{j}_i}(a)$$

are at most $\sum_i \sum_j P_i^j(a)$

$$\sum_{i \in S_{\mathrm{OPT}}} \sum_k R_k^{j_i^*}(a) \quad \text{and} \quad \sum_{i \in S_{\mathrm{EQ}}} R_k^{\hat{j}_i}(a)$$

are at most $\sum_i \sum_j R_i^j(a)$ and that

$$\sum_{i \in S_{\mathrm{OPT}}} \sum_k B_k^{j_i^*}(a_i, X_i(a)) \leq \sum_i \sum_j B_i^j(a_i, X_i(a))$$

Substituting these upper bounds and recalling that S_{OPT} and S_{EQ} partition the set of agents we get

$$u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \min(1, \lambda)\mathrm{OPT}(v) - (1+\mu_1) \sum_i \sum_j P_i^j(a) - (1+\mu_2) \sum_i \sum_j R_i^j(a) - \mu_3 \sum_i \sum_j B_i^j(a_i, X_i(a))$$

□

8 ROBUST WELFARE IN ROSCAS

We now combine the pieces developed in the previous sections to show two sets of welfare guarantees in roscas. First, we give a general result: reasonable (i.e. smooth) single-item mechanisms make approximately-optimal roscas. Second, we give a specific guarantees for first-price/descending-price and second-price/descending-price roscas by analyzing the single-item mechanism without an internalized seller and applying the general result.

Theorem 8.1. *Let M be an individually rational single-item auction which is weakly (λ, μ_1, μ_2)-smooth when C satisfies $C(0) = 0$ and $C'(x) \in [\alpha, \beta]$. If values are nonincreasing and value distributions are independent across agents the n-round round-robin sequential composition of M with an internalized seller has price of anarchy at most*

$$\frac{1 + \alpha + \mu_1 + (\alpha + \beta)\mu_2}{\alpha \min(1, \lambda)}$$

in any Bayes-Nash equilibrium without overpayment.

Proof. Because M is weakly (λ, μ_1, μ_2)-smooth with quasilinear bidders, Lemma 6.8 implies that with an internalized seller, it is also weakly $(\lambda, \mu_1, \mu_1, \mu_2)$-smooth before rebates, still with quasilinear bidders. Furthermore, Theorem 7.4 implies that the round robin composition of M is also $(\min(1, \lambda), 1 + \mu_1, 1 + \mu_1, \mu_2)$-smooth before rebates with quasilinear bidders. The Bayesian extension of Lemma 6.9 then implies that the price of anarchy of the mechanism is at most $(\alpha + \max(1 + \mu_1, 1 + \mu_1) + (\alpha + \beta)\mu_2)/\alpha \min(1, \lambda)$ □

Welfare in First-Price Roscas

A first-price rosca proceeds in the following way. Each round j, agents i who have not yet won an item submit sealed bids b_i^j. The highest bidder wins the item and is charged the highest bid $b_{(1)}$. Importantly, losers are rebated $b_{(1)}/(n-1)$. To give a welfare guarantee for the first-price rosca by Theorem 8.1, we only need to consider just the single-item first-price auction without an internalized seller. However, smoothness guarantees from existing work only hold for quasilinear agents. We adapt the standard bound as follows:

Lemma 8.2. *The single-item first price auction is $((1 - 1/e^{\beta})\beta^{-1}, 1)$-smooth with convex disutility for payments if $C'(x) \leq \beta$, $C(0) = 0$.*

Proof. We generalize the deviation from [46]. The highest valued agent, say index h, can deviate to submitting a randomized bid b'_h drawn from the distribution with density function $f(x) = \frac{1}{v_h - \beta \cdot x}$ and support $[0, (1 - 1/e^{\beta})v_h/\beta]$. The utility of the highest bidder from this deviation is:

$$\begin{aligned} u_h^{v_h}(b'_h, b_{-h}) &\geq \int_{\max_{i \neq h} b_i}^{\left(1-\frac{1}{e^{\beta}}\right)\frac{v_h}{\beta}} (v_h - C(x)) f(x) dx \\ &\geq \int_{\max_{i \neq h} b_i}^{\left(1-\frac{1}{e^{\beta}}\right)\frac{v_h}{\beta}} (v_h - \beta \cdot x) f(x) dx \\ &\geq \left(1 - \frac{1}{e^{\beta}}\right)\frac{v_h}{\beta} - \max_i b_i. \end{aligned}$$

Since the expected revenue is at least $\max_i b_i$, $\mathrm{OPT}(v) = v_h$ and the utilities and payments from all other agents are nonnegative the result follows. □

Applying Theorem 8.1, we obtain:

Corollary 8.3. *If values are nonincreasing, the first-price rosca has price of anarchy at most*

$$\frac{(\alpha + 2)\beta}{\alpha}\left(\frac{e^{\beta}}{e^{\beta} - 1}\right)$$

in any Bayes-Nash equilibrium without overpayment.

With quasilinear agents, $C(x) = x$ for all x, and Corollary 8.3 yields a price of anarchy of $3e/(e-1) \approx 4.75$. The guarantee degrades smoothly as agents' utilities become less linear in wealth.

Welfare in Second-Price Roscas

A second-price rosca proceeds in the following way. Each round j, agents i who have not yet won an item submit sealed bids b_i^j. The highest bidder wins the item and is charged the second-highest bid $b_{(2)}$. Importantly, losers are rebated $b_{(2)}/(n-1)$. We can also give guarantees for second-price roscas. As before, by Theorem 8.1, we need not consider more than one item or an internalized seller.

Lemma 8.4. *The single-item second-price auction is weakly* $(1, 0, 1)$*-smooth with convex disutility for payments if* $\alpha \leq C'(x) \leq \beta$, $C(0) = 0$ *and under no overbidding.*

Proof. Let v_h be the value of the highest bidder h. Then h can play a deviation $b'_h = v_h/\alpha$.

Case 1. If h wins with b'_h then

$$\begin{aligned} &u_h^{v_h}(b'_h, b_{-h}) = v_h - C(b_{(2)}) \\ &u_h^{v_h}(b'_h, b_{-h}) \geq v_h - B_{(2)}\left(b_{(2)}, X_{(2)}(b)\right) && \text{by definition} \end{aligned}$$

Case 2. If h loses with b'_h then

$$\begin{aligned}
b_{(2)} &\geq v_h/\alpha && \\
C(b_{(2)}) &\geq C(v_h/\alpha) && \text{by monotonicity of } C \\
C(b_{(2)}) &\geq v_h && \text{since } C(x) \geq \alpha x \text{ when } x \geq 0 \\
B_{(2)}(b_{(2)}, X_{(2)}(b)) &\geq v_h && \text{by definition} \\
u_h^{v_h}(b'_h, b_{-h}) &\geq v_h - B_{(2)}(b_{(2)}, X_{(2)}(b)) && \text{since utilities are nonnegative}
\end{aligned}$$

These are the only two cases, so since $\mathrm{OPT}(v) = v_h$ and since the utilities and willingness-to-pay from all other agents are nonnegative the result follows. □

Applying Theorem 8.1, we obtain:

Corollary 8.5. *If values are nonincreasing and assuming no overbidding, the second-price rosca has price of anarchy at most*

$$2 + \frac{1+\beta}{\alpha}$$

in any Bayes-Nash equilibrium without overbidding.

With quasilinear agents, $C(x) = x$ for all x, and Corollary 8.5 yields a price of anarchy of 3. The guarantee degrades smoothly as agents' utilities become less linear in wealth. A direct proof for a price of anarchy bound of $1 + 2\beta/\alpha$ for second-price roscas appeared in an earlier version of this work see Theorem A.2 in Appendix A.1 for this proof. However, the theory developed for weakly smooth mechanisms, like the second-price auction, in this paper allows us to prove a comparable constant-factor [6] approximation with significantly less work than that done for the proof of Theorem A.2.

9 DISCUSSION AND CONCLUSION

This work derives welfare guarantees for roscas which are robust in two senses: they are agnostic to the precise choice of value distributions and equilibria for those distributions, and they permit a variety of pot allocation protocols. This gives evidence that roscas can effectively coordinate joint saving by agents with investment opportunities which vary across time and between agents. The result provides further explanation for their wide use across many continents and more broadly demonstrates the power of mutual aid groups, informal financial organizations and other decentralized vehicles for community empowerment.

Our work suggests many further lines of inquiry where insights from algorithmic game theory may improve our understanding of these important financial institutions. Our modeling decisions focus on the ability of roscas to efficiently coordinate saving across time. Several authors have noted that roscas can serve as insurance as well [19, 34, 35]: agents with unanticipated, urgent financial needs can bid to obtain the pot earlier than they had planned. In general, understanding the efficiency of roscas as agents' values and incomes evolve stochastically over time, as well in the presence of heterogeneous wealth, poses significant technical challenges that might benefit from a variety of analytical perspectives.

One particular difficulty lies in the tension between allocative efficiency and wealth inequality. That is, agents with valuable investment opportunities may not be incentivized to bid aggressively, because low wealth causes them to value cash highly. This is particularly problematic when agents experience income shocks, a more common occurrence for economically vulnerable individuals

[6] Note these bounds agree for quasilinear bidders, but if $\alpha < \beta$ strictly, equivalent to $C(x) \neq mx + b$, the bound given by Corollary 8.5 is tighter.

[1]. One way rosca participants navigate this tension is altruism. Roscas often serve a dual role of community-building institutions. Consequently, agents tend to observe signals about each others shocks, and act with mutual aid in mind [36], [38]. Characterizing the way these phenomena interact, through game-theoretic, network science, or other approaches, seems key to building a broader theory for understanding how people self-organize to create opportunity.

ACKNOWLEDGMENTS

I would like to thank Professors Rediet Abebe (U.C Berkeley) and Sam Taggart (Oberlin College) for their many meetings and endless patience with my many questions. I would like to thank Professor Adam Eck (Oberlin College) for his advising and keeping me on track with the thesis process. I would like to thank God, my Mum in heaven, my Dad and my Brother to who I owe any of my successes. I would also like to thank my friends in the Computer Science and Math departments particularly Tumas Rackaitis and Kyndelle Johnson, and the MD4SG Inequality Working Group whose many conversations and insightful questions/comments inspired many of the ideas in this thesis.

REFERENCES

[1] Rediet Abebe, Jon M Kleinberg, and S Matthew Weinberg. 2020. Subsidy Allocations in the Presence of Income Shocks.. In *AAAI*. 7032–7039.

[2] W Adams Dale and Marie L Canavesi. 1992. Rotating savings and credit associations in Bolivia. *Adams and Fitchett, Informal Finance in Low-Income Countries* (1992).

[3] Goski Alabi, Joshua Alabi, and Stephen Tei Akrobo. 2007. The Role Of Susu A Traditional Informal Banking System In The Development Of Micro And Small Scale Enterprises (MSEs) In Ghana. *International Business & Economics Research Journal (IBER)* 6, 12 (2007).

[4] Ernest Amankwah, Fritz Augustine Gockel, Osei-Assibey Eric, and Alice Nubuor. 2019. Pareto superior dimension of rotating savings and credit associations (ROSCAs) in Ghana: Evidence from Asunafo North Municipality of Ghana. *Journal of Economics Library* 6, 4 (2019), 287–309.

[5] Siwan Anderson and Jean-Marie Baland. 2002. The economics of roscas and intrahousehold resource allocation. *The Quarterly Journal of Economics* 117, 3 (2002), 963–995.

[6] Shirley Ardener. 1964. The comparative study of rotating credit associations. *The Journal of the Royal Anthropological Institute of Great Britain and Ireland* 94, 2 (1964), 201–229.

[7] Shirley Ardener and Sandra Burman. 1995. Money-go-rounds: The importance of rotating savings and credit associations for women. (1995).

[8] Dejene Aredo. 2004. Rotating savings and credit associations: characterization with particular reference to the Ethiopian Iqqub. *Savings and Development* (2004), 179–200.

[9] Jean-Marie Baland, Catherine Guirkinger, and Renate Hartwig. 2019. Now or later? The allocation of the pot and the insurance motive in fixed roscas. *Journal of Development Economics* 140 (2019), 1–11.

[10] Abhijit V Banerjee and Esther Duflo. 2010. Giving credit where it is due. *Journal of Economic Perspectives* 24, 3 (2010), 61–80.

[11] Michaela Beck. 2012. *Impact of Accumulating Savings and Credit Associations on Child Well-Being*. Ph.D. Dissertation. masters thesis.

[12] Timothy Besley and Stephen Coate. 1995. Group lending, repayment incentives and social collateral. *Journal of development economics* 46, 1 (1995), 1–18.

[13] Timothy Besley, Stephen Coate, and Glenn Loury. 1993. The economics of rotating savings and credit associations. *The American Economic Review* (1993), 792–810.

[14] Timothy Besley, Stephen Coate, and Glenn Loury. 1994. Rotating savings and credit associations, credit markets and efficiency. *The Review of Economic Studies* 61, 4 (1994), 701–719.

[15] Sushil Bikhchandani. 1999. Auctions of heterogeneous objects. *Games and Economic Behavior* 26, 2 (1999), 193–220.

[16] FJA Bouman. 1995. Rosca: On the Origin of the Species/Rosca: sur L'Origine du Phenomène. *Savings and Development* (1995), 117–148.

[17] Frits JA Bouman. 1995. Rotating and accumulating savings and credit associations: A development perspective. *World development* 23, 3 (1995), 371–384.

[18] Kenneth P Brevoort, Philipp Grimm, and Michelle Kambara. 2015. Data point: Credit invisibles. *Consumer financial protection bureau* (2015), 1–37.

[19] Charles W Calomiris and Indira Rajaraman. 1998. The role of ROSCAs: lumpy durables or event insurance? *Journal of development economics* 56, 1 (1998), 207–216.

[20] Adélie Chevée. 2021. Mutual Aid in north London during the Covid-19 pandemic. *Social Movement Studies* (2021), 1–7.

[21] Stijn Claessens. 2006. Access to financial services: A review of the issues and public policy objectives. *The World Bank Research Observer* 21, 2 (2006), 207–240.

[22] Catherine Cross. 1987. Informal lending: do-it-yourself credit for black rural areas. *Indicator South Africa* 4, 3 (1987), 87–92.

[23] Constantinos Daskalakis, Paul W Goldberg, and Christos H Papadimitriou. 2009. The complexity of computing a Nash equilibrium. *SIAM J. Comput.* 39, 1 (2009), 195–259.

[24] Asli Demirguc-Kunt, Leora Klapper, Dorothe Singer, Saniya Ansar, and Jake Hess. 2018. *The Global Findex Database 2017: Measuring Financial Inclusion and the Fintech Revolution.* The World Bank. https://doi.org/10.1596/978-1-4648-1259-0

[25] David Easley, Jon Kleinberg, et al. 2010. *Networks, crowds, and markets*. Vol. 8. Cambridge university press Cambridge.

[26] Marcel Fafchamps and Susan Lund. 2003. Risk-sharing networks in rural Philippines. *Journal of development Economics* 71, 2 (2003), 261–287.

[27] Hanming Fang, Rongzhu Ke, and Li-An Zhou. 2015. *Rosca meets formal credit market.* Technical Report. National Bureau of Economic Research.

[28] Michal Feldman, Nicole Immorlica, Brendan Lucier, Tim Roughgarden, and Vasilis Syrgkanis. 2016. The price of anarchy in large games. In *Proceedings of the forty-eighth annual ACM symposium on Theory of Computing*. 963–976.

[29] LLC Gurobi Optimization. 2021. Gurobi Optimizer Reference Manual. http://www.gurobi.com

[30] Jon Haveman. 2015. Generating Household Wealth and Financial Access Estimates for Local Geographies. *Technical Documentation for the Corporation for Enterprise Development* (2015), 1–14.

[31] Nathan Jun and Mark Lance. 2020. Anarchist Responses to a Pandemic: The COVID-19 Crisis as a Case Study in Mutual Aid. *Kennedy Institute of Ethics Journal* 30, 3 (2020), 361–378.

[32] Francois I Kabuya. 2015. The rotating savings and credit associations (ROSCAs): Unregistered sources of credit in local communities. *IOSR Journal of Humanities and Social Science* 20, 8 (2015), 95–98.

[33] Thomas Kesselheim and Bojana Kodric. 2018. Price of Anarchy for Mechanisms with Risk-Averse Agents. In *45th International Colloquium on Automata, Languages, and Programming (ICALP 2018)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

[34] Stefan Klonner. 2001. How Roscas perform as insurance. (2001).

[35] Stefan Klonner. 2003. Rotating savings and credit associations when participants are risk averse. *International Economic Review* 44, 3 (2003), 979–1005.

[36] Stefan Klonner. 2008. Private information and altruism in bidding ROSCAs. *The Economic Journal* 118, 528 (2008), 775–800.

[37] Jens Kovsted and Peter Lyk-Jensen. 1999. Rotating savings and credit associations: the choice between random and bidding allocation of funds. *Journal of Development Economics* 60, 1 (1999), 143–172.

[38] Getachew Mequanent. 1996. The Role of Informal Organizations in Resettlement Adjustment Process: A Case Study of Iqubs, Idirs, and Mahabers in the Ethiopian Community in Toronto. *Refuge: Canada's Journal on Refugees* (1996), 30–40.

[39] Debra Mesch, Una Osili, Tessa Skidmore, Jon Bergdoll, Jacqueline Ackerman, and Jeannie Sager. 2020. COVID-19, Generosity, and Gender: How Giving Changed During the Early Months of a Global Pandemic. (2020).

[40] Noam Nisan, Tim Roughgarden, and Eva Tardos. 2007. *Algorithmic Game Theory.* Cambridge University Press. https://doi.org/10.1017/CBO9780511800481

[41] Kanayo Ogujiuba, Fadila Jumare, and Nancy Stiegler. 2013. Challenges of microfinance access in Nigeria: Implications for entrepreneurship development. (2013).

[42] Shaik Abdul Majeeb Pasha and Ato Debalke Delcho Dayrra. 2016. Role of 'Iqqubs' in Private Business Start Up and Development of Smes with Reference to Arba Minch, Ethiopia. *Journal of Social Welfare and Management* 8, 2 (2016).

[43] Kristiano Raccanello and Jayant Anand. 2009. Health expenditure financing as incentive for participation in ROSCAs. *Revista Desarrollo y Sociedad* 64 (2009), 173–206.

[44] Tim Roughgarden. 2009. Intrinsic robustness of the price of anarchy. In *Proceedings of the forty-first annual ACM symposium on Theory of computing*. 513–522.

[45] Tim Roughgarden, Vasilis Syrgkanis, and Eva Tardos. 2017. The price of anarchy in auctions. *Journal of Artificial Intelligence Research* 59 (2017), 59–101.

[46] Vasilis Syrgkanis and Eva Tardos. 2013. Composable and efficient mechanisms. In *Proceedings of the forty-fifth annual ACM symposium on Theory of computing*. 211–220.

[47] Penny Travlou. 2020. Kropotkin-19: A Mutual Aid Response to COVID-19 in Athens. *Design and Culture* (2020), 1–10.

APPENDIX

A OMITTED PROOFS AND RESULTS

A.1 Welfare in Second-Price Roscas

Our analysis will proceed in the spirit of the weak smoothness framework of [46]: for each bidder, we will construct a deviation strategy which exhibits a tradeoff between their utility and the bids of other agents in the auction. Our tradeoff will hold in every deterministic profile of actions and for every value profile. We then show how to extend the deviation to derive a welfare result in Bayes-Nash equilibrium. In what follows, recall that actions in a rosca are contingency plans over bids b_i^j in each round.

Lemma A.1. *For any value profile v and action profile a of a second-price rosca, there exists a deviation action $a_i^*(v, a_i)$ for each agent i such that the following inequality holds:*

$$\sum_i u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq OPT(v) - 2\sum_j C\big(b_{(1)}^j\big), \tag{12}$$

where $b_{(1)}^j$ denotes the highest in round j under action profile a.

The proof will emulate the closure of smooth mechanisms under simultaneous composition in [46]: we will have each agent i emulate their strategy a_i until the item they would win in the optimal assignment, then target the item they would win in the optimal assignment. A key distinction is that if i wins before their targeted round, they are disqualified from future rounds. Under nonincreasing values, we show that this doesn't hurt much.

Proof. An optimal allocation given value profile v is an assignment from agents i to items j_i^*. For values v and action profile a, we construct a deviation $a_i^*(v, a_i)$ for each agent i in the following way. Agent i simulates their equilibrium strategy up until round $j_i^* - 1$. In round j_i^*, they bid $C^{-1}(v_i^{j_i^*})$ and then 0 in all subsequent rounds. If, in simulating a_i, agent i wins in some round $\hat{j}_i < j_i^*$ they similarly bid 0 in all subsequent rounds including j_i^*. Let S_{EQ} denote the set of agents whose deviations cause them to win in some $\hat{j}_i < j_i^*$, and let S_{OPT} denote the set of agents who don't win before j_i^* and hence are deviate in round j_i^*. Note that S_{EQ} and S_{OPT} together partition the set of agents.

For each agent $i \in S_{\text{OPT}}$, we may consider two cases, based on whether their deviation causes them to win or lose in round j. If they win, let i' be the highest bidder other than i. Since i incurs nonnegative utility outside of round j_i^*, we have that $u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq v_i^{j_i^*} - C(b_{i'})$. We can therefore write:

$$u_i^{v_i}(a_i^*(v, a_i), a_{-i}) + C\big(b_{(1)}^j\big) \geq u_i^{v_i}(a_i^*(v, a_i), a_{-i}) + C(b_{i'}) \geq v_i^{j_i^*}.$$

If i instead loses with their deviation in round j_i^*, then we instead have $b_{i'} \geq C^{-1}(v_i^{j_i^*})$. Since C is increasing, we obtain

$$C(b_{(1)}^{j_i^*}) \geq C(b_{i'}) \geq v_i^{j_i^*}.$$

In either case, we have shown that for all $i \in S_{\text{OPT}}$,

$$u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq v_i^{j_i^*} - C\big(b_{(1)}^{j_i^*}\big). \tag{13}$$

For each agent $i \in S_{\text{EQ}}$, meanwhile, note that their payments are nonpositive outside round $\hat{j}_i$. Since utility is value minus disutility for payments, we can write:

$$u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq v_i^{\hat{j}_i} - C\big(b_{(2)}^{\hat{j}_i}\big) \geq v_i^{j^*} - C\big(b_{(1)}^{\hat{j}_i}\big), \tag{14}$$

where the second inequality follows from the nonincreasing values assumption and the fact that $b^{\hat{j}_i}_{(1)} \geq b^{\hat{j}_i}_{(2)}$.

Summing over all agents, we obtain

$$\sum_i u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq \mathrm{OPT}(v) - \sum_{j \in S_{\mathrm{OPT}}} C\big(b^{j_i^*}_{(1)}\big) - \sum_{j \in S_{\mathrm{EQ}}} C\big(b^{\hat{j}_i}_{(1)}\big).$$

Since each round j is j_i^* for at most one agent i and $\hat{j}_i$ for at most one agent i. we may weaken the above inequality to obtain (12). □

We are now in a position to reason about the second-price rosca in Bayes-Nash equilibrium. The main result of this section is the following.

Theorem A.2. *Assuming no overbidding and nonincreasing values, the price of anarchy for the second-price rosca is at most* $1+2\beta/\alpha$ *in Bayes-Nash equilibrium with independently distributed values across agents.*

The guarantee in Theorem A.2 depends on the shape of the disutility function C, but degrades smoothly away from a price of anarchy of 3 as the ratio of C's slopes varies away from 1. The proof follows the "bluffing" template of [46]. Note that we require independence across agents, but not across the values for a particular agent. In fact, the nonincreasing values assumption rules the latter sort of independence out almost always.

Proof. Let s be a Bayes-Nash equilibrium. For a bidder i, consider the following "bluff" deviation which lowerbounds their expected equilibrium utility: have them sample a value profile w according to agents' distributions, and play $a_i^*((v_i, w_{-i}), s_i(w_i))$, where a_i^* is as defined in Lemma A.1. We may therefore lower bound ex ante equilibrium expected utility $\mathbb{E}_v[u_i^{v_i}(s(v))]$ as:

$$\begin{aligned}&\mathbb{E}_{v,w}[u_i^{v_i}(a_i^*((v_i, w_{-i}), s_i(w_i)), s_{-i}(v_{-i}))] \\ &\qquad = E_{v,w}[u_i^{w_i}(a_i^*((w), s_i(v_i)), s_{-i}(v_{-i}))],\end{aligned}$$

where the latter inequality follows from exchanging the roles of v and w, which is valid because of independence. Summing over agents and applying Lemma A.1, we have:

$$\begin{aligned}&\mathbb{E}_{v,w}\Big[\sum\nolimits_i u_i^{w_i}(a_i^*((w), s_i(v_i)), s_{-i}(v_{-i}))\Big] \\ &\geq E_{v,w}[\mathrm{OPT}(w)] - E_{v,w}\Big[2\sum\nolimits_j C\big(b^j_{(1)}\big)\Big] \\ &\geq E_v[\mathrm{OPT}(v)] - 2E_v\Big[\sum\nolimits_i v_i \cdot X_i(s(v))\Big] \end{aligned} \tag{15}$$

where the second inequality follows from the no-overbidding assumption, and the third from the fact that payments are redistributed.

To obtain a welfare bound, note that expected utilities can be written as

$$E_v[u_i^{v_i}(s(v))] = E_v\big[v_i \cdot X_i(s(v)) - \sum\nolimits_j C(P_i^j(s(v))).\big]$$

Therefore we have

$$\frac{E_v\big[\sum_i v_i \cdot X_i(s(v))\big]}{E_v\big[\sum_i u_i^{v_i}(s(v))\big]} \leq \frac{E_v\big[\sum_i \sum_j \max(C(P_i^j(s(v))), 0)\big]}{E_v\big[\sum_i \sum_j -\min(C(P_i^j(s(v))), 0)\big]}, \tag{16}$$

obtained by subtracting $E_v[v_i \cdot X_i(s(v)) - \sum_j \max(C(P_i^j(s(v))), 0).]$ from the numerator and denominator of the lefthand side, which is nonnegative by the no overbidding assumption. With an internalized seller, we have

$$\sum_i \sum_j -\min(P_i^j(s(v)), 0) = \sum_i \sum_j \max(P_i^j(s(v)), 0).$$

Since $C(0) = 0$ and $C'(x) \in [\alpha, \beta]$, the righthand side of (16) is at most β/α. Combining with (15), we obtain:

$$\mathbb{E}_v\Big[\sum_i u_i^{v_i}(s(v))\Big] \geq E_v[\mathrm{OPT}(v)] - \tfrac{2\beta}{\alpha} E_v\Big[\sum_i u_i^{v_i}(s(v))\Big].$$

The result follows from rearranging. □

Note that for dynamic games under incomplete information as in our model, the usual equivalence of equilibria between sealed-bid and ascending-price mechanisms does not necessarily hold. However, the proof of Theorem A.2 remains valid, because of its reliance on deviation bids. A deviation in the sealed-bid version of the game corresponds to a deviation in the ascending-price game via the usual correspondence, and the no overbidding assumption becomes an assumption that players drop out before they would begin incurring negative utility.

A.2 No Overbidding in Strongly Smooth Mechanisms

Proof Sketch. Rebates in each round have to be less than $C^{-1}(v_{w_j}(x_{w_j}))$ where w_j denotes the agent who won the item in round j. Since our mechanisms are individually rational w_j is the only one making positive payments each round and thus they have to pay less than $C^{-1}(v_{w_j}(x_{w_j}))$ to avoid negative utility. Thus if an agents is overbidding to get higher rebates in future rounds this only works if their overbid causes them to win in strongly smooth mechanisms but as we showed earlier the winning agent in each round is never overbidding. □

B LINEAR PROGRAMMING FOR AFTER-MARKET

This is the linear program used to construct worst case examples for swap-stable assignments with non-increasing values.

$$\begin{array}{llll}
\text{maximize} & \displaystyle\sum_{i=1}^{n} v_{ii} - \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}v_{ij} & & \\
\text{subject to} & \displaystyle\sum_{i=1}^{n} v_{ii} & \geq \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}v_{ij}, & i = 1, \dots, n \\
& v_{ij} & \geq v_{ij'}, & \text{if } j < j', \forall i \in 1, \dots, n \\
& a_{ij}v_{ij} + a_{i'j'}v_{i'j'} & \geq a_{ij}v_{ij'} + a_{i'j'}v_{i'j}, & \forall i, j, i', j' \in 1, \dots, n \text{ if } a_{ij}, a_{i'j'} \geq 1 \\
& v_{ij} & \leq 1 & \forall i, j \\
& v_{ij} & \geq 0 & \forall i, j
\end{array}$$

- v_{ij} are the LP decision variables, and are person i's value for getting assigned position j
- a_{ij} are constant indicators for whether person i is assigned position j

The python implementation using Gurobi [29] for the $x(1) = 3, x(2) = 1, x(3) = 2$ swap-stable assignment is displayed on the following pages.

```
[1]: import numpy as np
     import pandas as pd
     import gurobi as gp
     from gurobipy import GRB
```

```
[9]: try:
         #creating model
         m = gp.Model("3cycle")


         #create assigment constants
         #aij is 1 if person i has been assigned position j, 0 otherwise
         a11, a12, a13, a21, a22, a23, a31, a32, a33 = 0, 0, 1,  1, 0, 0,  0, 1, 0 ␣
      ↪#3 1 2

         #create decision variables
         #vij is person i's value for being assigned position j
         v11 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v11")
         v12 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v12")
         v13 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v13")
         v21 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v21")
         v22 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v22")
         v23 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v23")
         v31 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v31")
         v32 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v32")
         v33 = m.addVar(lb=0.0,ub=1.0,vtype=GRB.CONTINUOUS,name="v33")


         #create objective function
         #we impose that the optimal assigment is person i gets assigned position i
         #we want to maximize the difference in welfare between the optimal␣
      ↪assigments and our assigment
         m.setObjective(v11+v22+v33 - (a11*v11 + a12*v12 + a13*v13 + a21*v21 +␣
      ↪a22*v22 + a23*v23 + a31*v31 + a32*v32 +  a33*v33), GRB.MAXIMIZE)



         #impose the optimal assigment is person i gets position i
         m.addConstr(v11+v22+v33 >= a11*v11 + a12*v12 + a13*v13 + a21*v21 + a22*v22 +␣
      ↪a23*v23 + a31*v31 + a32*v32 +  a33*v33,"o1")

         #nonincreasing values constraints for agents
         m.addConstr(v11 >= v12,"m11")
         m.addConstr(v12 >= v13,"m12")
         m.addConstr(v21 >= v22,"m21")
         m.addConstr(v22 >= v23,"m22")
         m.addConstr(v31 >= v32,"m31")
```

```
    m.addConstr(v32 >= v33,"m32")

    #swap stability constraints v_ija_ij + v_i'j'a_i'j' >= v_ij'a_ij' +
↪v_i'ja_i'j for our specific assigment

    #3 1 2
    m.addConstr(v13 + v21 >= v11 + v23 ,"s1")
    m.addConstr(v13 + v32 >= v12 + v33 ,"s2")
    m.addConstr(v21 + v32 >= v22 + v31 ,"s3")



    # Optimize model
    m.optimize()

    #print solutions
    for v in m.getVars():
        print('%s %g' % (v.varName, v.x))

    print('Obj: %g' % m.objVal)




#error handling
except gp.GurobiError as e:
    print('Error code ' + str(e.errno) + ': ' + str(e))

except AttributeError:
     print('Encountered an attribute error')
```

```
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (mac64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 10 rows, 9 columns and 30 nonzeros
Model fingerprint: 0xd2b25932
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 7 rows and 6 columns
Presolve time: 0.00s
Presolved: 3 rows, 3 columns, 7 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    2.0040000e+00   2.004000e+00   0.000000e+00      0s
Extra simplex iterations after uncrush: 1
```

```
       3    1.0000000e+00   0.000000e+00   0.000000e+00      0s

Solved in 3 iterations and 0.01 seconds
Optimal objective  1.000000000e+00
v11 1
v12 0
v13 0
v21 1
v22 1
v23 0
v31 0
v32 0
v33 0
Obj: 1
```