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An Analysis of Education Subsidy in the presence of Fertility Decisions, Human Capital Accumulation, and Spillovers

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The paper hypothesizes that the level of aggregate human capital in an economy affects the overall returns of an individual's human capital accumulation and fertility decision. However, the positive externalities from aggregate human capital are not internalized by households in their investment decisions. Using an overlapping-generations model, specify the household's optimization problem in a rural developing country where parents have an old-age support motive that underlies the benefits of having and educating children. The optimal decision of parents is then contrasted with that of the social planner. They differ in that the social planner internalizes the externalities of aggregate human capital on the single consumption good production. The difference between investment decisions yields the country's optimal education subsidy. The model is tested empirically on secondary education across 59 countries using difference-in-differences and OLS regressions.

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1 Introduction

Human capital plays a central role in shaping the growth of economies and is a crucial determinant of a person's and a nation's productivity. Öztürk (2001) even labeled the 20th century as the "Age of Human Capital" since a country's standard of living now is primarily determined by how well it succeeds in developing the skills, utilizing the knowledge, and furthering the health and education of its citizens. For developing countries, the most prevalent method of accumulating human capital is education. As the benefits of education are manifold and unquantifiable, including promoting competitiveness, slowing down birth rates, and increasing per capita income growth, it is evident that there are positive externalities to increasing aggregate human capital. This is proven by the prioritizing of education investment initiatives by all parties involved, from the Millennium Development Goal's target for universal primary education to localized plans from governments like Wawasan 2020 (Vision 2020) in Malaysia. The eagerness to speed up the original rate of education investment clearly shows an implicit understanding that, if left to their own accord, the citizens of these countries lack the incentive to invest in human capital at the rate the local government or international aid and development agencies would consider ideal. However, faced with the small budgets that developing countries are usually working with, what should the subsidy look like? How much is necessary to generate the results that they want to see?

The purpose of my paper is twofold. First, it is to present a costs and benefits analysis that is modeled after a rural developing economy. This analysis applies to two parties: the average household in the economy and a hypothetical social planner who is omnipotent. Since the households do not internalize the positive externalities of aggregate human capital, their choices of investment in the human capital of their children and the number of children to have will be suboptimal. The social planner, however, will see all the effects of human capital and fertility decisions in the economy and take them into account. Comparing both decisions, an optimal education subsidy can be developed. Internalizing this policy instrument, I analyze the change in parameter relationships as the model economy funds the subsidy with a payroll tax. Furthermore, I test the general implications of the model empirically across 59 countries for real world significance using difference-in-difference analysis and OLS regressions.

The literature on human capital, fertility, and growth is substantial. In models of endogenous growth, Becker, Murphy, and Tamura (1990) as well as Galor and Weil (2000) frame fertility decisions based on the quantity-quality tradeoff between the number of children and the education of each child. They assume increasing returns to human capital in their model, indicating its positive externality. Morand (1999) presents a similar growth model with an old-age support motive that is more reflective of rural developing economies. Positive externalities relating to education is also a subject that has been widely studied (Moretti, 2004; Mueller, 2007; Hall, 2006), although there is still no consensus on its quantification. I have not found treatments of optimal subsidies for education in the economic growth literature. However, the framework that Goodfriend and King (2001), as well as Khan, King, and Wolman (2000) created for determining optimal monetary tax policies provides good analogs.

The contributions of this work will be to create a theoretical growth model with the same quantity-quality tradeoff between the number of children and the education of each child that is representative of a rural developing economy. This means that the model will incorporate the old-age support motive and imperfect credit markets. This model will then be solved from the household's and then the social planner's perspective, since the household does not take into account the effect of its decisions on the aggregate stock of human capital. Becker, Murphy, and

Tamura (1990) have a solid theoretical framework that has influenced much of this model. However, in their models they assumed that households internalize the human capital externality and that parents possess an altruistic utility function. This may not be the case with developing countries where the old-age support motive is prevalent. Morand starts out with a rural developing economy in mind but did not take into account the fact that households do not internalize the externality. Moreover, he also assumes that parent's education levels only affect their investment in children positively. They do not take into account the opportunity cost of raising children; the opportunity cost is related to market wages and increases with human capital. The externality papers focused more on testing for the existence of externalities, and the concentration is on tertiary education. Since this paper is modeling a rural developing economy, my work deals with lower levels of education rather than tertiary education. This is because education is cumulative and developing countries, often faced with low enrollment numbers at the primary and secondary levels, have to focus on increasing these enrollment rates before moving on to tertiary education. The optimum subsidy measure shows a basic framework of the relationships between different factors in the economy. If developed further, it can be used to examine the effectiveness of education initiatives in different countries.

The suboptimization of the household is confirmed by the proving of a lower steady state human capital investment level and a higher steady state fertility level compared to the social planner's levels. The optimal subsidy confirms original intuition in terms of its relationships with different parameters in the model. The incorporation of a payroll tax changes some of the dynamics within the first optimal subsidy specification compared to the second. This is due to the way the tax is structured. On the empirical side, the difference-in-differences analysis shows an effective education policy change between the lower income groups and the high income group. The regression results confirm most intuitions but also highlighted some problems with some of the selected proxies, especially the proxy for the secondary education subsidy. This will be discussed further in the paper.

The paper is organized as follows. Section 2 explains the theoretical model and describes the decisions of the household and the social planner, establishing the need for an education subsidy. Section 3 follows with the incorporation of the policy instrument into the theoretical economy. The general validity of this model will then be tested in Section 4 empirically and Section 5 will be the conclusion.

2 The Model

2.1 The Decentralized Economy

The model employs the overlapping-generations framework to construct the individual household's consumption decision. Individuals live for three periods in this model but are productive in only one. In the first period, they are born and will make no choices because they consume a portion of their parents' production. This model presupposes that the only costs associated with having or educating offspring are production goods[1]. In a sense, parents will pay to have children and to give them human capital. In the second period, individuals become productive and supply one unit of labor inelastically. They pay out a portion of their wage to their parents, invest in their offspring and their offspring's education, and spend the rest on their own consumption. In the third period, individuals make no decisions but are assumed to consume a portion of all their children's' wages. These individuals are supported by their children and this support system is assumed to continue from generation to generation, reflective of the old-age

[1] Some papers use time as an input to having children, like Becker, Tamura and Murphy (1990).

support motive prevalent in many developing countries[2]. Developing economies typically have imperfect credit markets and so it is assumed that there is no way of transferring income from one period to the next via financial transactions. There are no savings and all goods are perishable by the end of each period. Children and their education will be the only investment transferable from period to period. It is assumed that there is no heterogeneity amongst the households in the economy. The model uses households as a production unit[3].

In period *t*, the household is productive and receives the marginal product of their labor as wage, w_t. A traditional, exogenous, and fixed fraction, θ, from one's wage, is given to parents as old-age support[4]. The portion of the income allocated to raising children is $(\phi h_{t-1}^{\varepsilon} + \tau h_t)n_t$. ϕ is the fixed cost of having a child, which includes food, clothing and other basic expenses ($0 < \phi < 1$). h_{t-1}^{ε} is the parents' human capital determined from the period before which translates into the opportunity cost of having children with returns ε ($0 < \varepsilon < 1$). As the parents' wages increase as a function of their education, it will be costlier to have children since they will be taking time away from earning more money. The per unit cost of education is τ, a positive parameter ($0 < \tau < 1$). Lastly, h_t is the education level of each child which translates directly into its cost, and n_t is the number of children. The household's consumption function at period *t* is:

$$c_t = \left(1 - \theta - (\phi h_{t-1}^{\varepsilon} + \tau h_t)n_t\right)w_t \tag{1}$$

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[2] This support system can be attributed to the cultural norm and also to the lack of a social security system or pension fund plan, both of which are reflective of rural developing economies.

[3] It is acknowledged that there is a discrepancy between households in the first period having children, and the children individually becoming households in the second period and having children on their own. However, the complex modeling of this discrepancy is not pursued in this paper.

[4] Leibenstein (1975) outlines the argument that the decision to have children is based on the old-age support motive as part of an informal trading system. There is other evidence that support the individual family as the dominant institution of old-age support in developing countries (Jiang, 1994; Jensen, 1990; Dharmalingam, 1994). Nugent (1985) presents a helpful and objective oversight on the importance of this motive as part of endogenous growth models.

Because c_t must be non-negative, the constraint $\theta + (\phi h_{t-1}^{\varepsilon} + \tau h_t)n_t \leq 1$ must hold.

In the period t+1, the individual is no longer working and will be consuming the fixed share θ from the total of their children's wages.

$$c_{t+1} = \theta w_{t+1} n_t \tag{2}$$

The optimization problem the household is facing is then to choose between number of children to have, n_t and the human capital investment for each child, h_t, in order to maximize utility from their consumption both when productive and during retirement. With β being the inter-temporal elasticity of substitution, the household optimization problem to be solved, subject to the budget constraints above, is

$$\max_{c_t, c_{t+1}} u(c_t) + \beta u(c_{t+1}) \tag{3}$$

The economy's production function will include only human capital as an input. The reason for this is firstly for simplicity and secondly because all physical capital is in some shape or form created, designed, and operated by human capital. In that sense it is encompassed in this model. Output is thus given by,

$$f_t(H, h, n) = H_{t-1}^{\delta} h_{t-1}^{\gamma} n_{t-1} \tag{4}$$

H_{t-1} is the aggregate level of human capital embedded in current productive members of workforce, h_{t-1} is the individual's human capital, and n_{t-1} is the size of the productive population. The individual human capital term is lagged by one period because the education process takes place one period earlier for all working individuals. The return to individual human capital, γ is a positive fraction ($0 < \gamma < 1$). This includes exogenous factors like land. The return to the aggregate effects of human capital is δ, a positive fraction. The aggregate level of human capital, H_{t-1}, is the product of each individual's human capital, multiplied by the number of children raised last period as well as all the number of identical households who raised and educated these children in the previous period, $H_{t-1} = n_{t-1}h_{t-1}n_{t-2}$.

Assuming the labor market is in equilibrium and the wage is the marginal product of labor,

$$w_t = H_{t-1}^{\delta} h_{t-1}^{\gamma} \tag{5}$$

For simplicity, I use log functions for the consumption utilities.

$$u(c_t) = \ln c_t$$
$$u'(c_t) = \frac{1}{c_t}$$

Using (1) and (2), the first order condition for the optimal human capital investment is,

$$u'(c_t) = \beta \frac{\theta\gamma H_{t-1}^{\delta} h_t^{\gamma-1} n_t}{\tau H_{t-1}^{\delta} h_{t-1}^{\gamma} n_t} u'(c_{t+1})$$

After reworking the equation,

$$h_t = \frac{\beta\gamma}{\tau(1+\beta\gamma)}\left[\frac{(1-\theta)}{n_t} - \phi h_{t-1}^{\varepsilon}\right] \tag{6}$$

Taking the derivative of h_t with respect to n_t,

$$\frac{dh_t}{dn_t} = -\frac{\beta\gamma}{\tau(1+\beta\gamma)}\left[\frac{(1-\theta)}{n_t^2}\right]$$

There is a definite negative relationship between human capital accumulation and fertility. Unsurprisingly, the cost of education, τ, also affects human capital accumulation negatively. A higher cost of education leads to lower investment in education of offspring. The increase of the allocation to the productive household's parents, θ, will decrease human capital accumulation because it decreases the overall portion of income leftover for educating and rearing their own children. As well, the higher the human capital of the parents, h_{t-1}^{ε}, the more costly is rearing children.

The first order condition for the fertility decision is,

$$u'(c_t) = \beta \frac{\theta H_t^{\delta} h_t^{\gamma}}{(\phi h_{t-1}^{\varepsilon} + \tau h_t) H_{t-1}^{\delta} h_{t-1}^{\gamma}} u'(c_{t+1})$$

Reworking the equation,

$$n_t = \frac{\beta}{(1+\beta)} \left[\frac{(1-\theta)}{(\phi h_{t-1}^{\varepsilon} + \tau h_t)} \right] \tag{7}$$

The derivative with respect to h_t from this first order condition is,

$$\frac{dn_t}{dh_t} = -\frac{\beta\tau(1-\theta_p)}{(1+\beta)(\phi h_{t-1}^{\varepsilon} + \tau h_t)^2}$$

This also confirms the inverse relationship between fertility and human capital accumulation. The cost of having children, ϕ, the opportunity cost of the parents, h_{t-1}^{ε}, and the fraction paid out to the retired grandparents, θ, all have a negative correlation with fertility. Combining (6) and (7) to get optimal choices for both fertility and human capital accumulation,

$$\tilde{n}_t = \frac{\beta}{(1+\beta)} \left[\frac{(1-\theta)(1-\gamma)}{\phi h_{t-1}^{\varepsilon}} \right] \tag{8}$$

The optimal fertility decision for households, $\tilde{n}_t$, confirms the initial intuition. The higher the cost of rearing children, the lower the number of children the parents will choose. As well, an increase in the opportunity cost of parent’s time will also lower optimal fertility. As the returns to individual human capital, γ, increases this means that human capital investment will be favored and so optimal fertility will drop. Lastly, the fraction paid out to retired parents has an inverse relationship with the household's fertility decision since it decreases overall household income. The inter-temporal elasticity of substitution has a more ambiguous effect on the optimal fertility decision.

$$\tilde{h}_t = \frac{\gamma}{1-\gamma}\frac{\phi}{\tau}h_{t-1}^{\varepsilon} \tag{9}$$

The optimal human capital accumulation above, $\tilde{h}_t$, depends positively on the cost of having children and negatively on the cost of education. An increase in cost of having children will mean that parents will substitute having more children for educating the children further. The optimal human capital investment is also positively related to the parents' education. Because higher parental human capital translates into higher opportunity costs for having children, highly educated parents will invest in the quality of children rather than quantity.

From (9), it is possible to find the steady state for human capital accumulation, h^*_i. By equating $h_t = h_{t-1}$,

$$h^*_i = \left[\frac{\gamma}{1-\gamma}\frac{\phi}{\tau}\right]^{\frac{1}{1-\varepsilon}} \tag{10}$$

This steady state shows that the economy converges towards a certain value of human capital. To find the steady state for fertility, h^*_i is substituted into (8),

$$n^*_i = \frac{\beta(1-\theta)}{(1+\beta)}\left(\frac{(1-\gamma)}{\phi}\right)^{\frac{1}{1-\varepsilon}}\left(\frac{\tau}{\gamma}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{11}$$

Increasing levels of human capital over time would need the following,

$$h_t \geq h_{t-1}$$

$$h_{t-1} \leq \left[\frac{\gamma}{1-\gamma}\frac{\phi}{\tau}\right]^{\frac{1}{1-\varepsilon}}$$

This shows the ceiling level of human capital the parents need to have in order to educate their offspring more than themselves. When the cost of having children, ϕ, is very low, this means a low threshold level of h_{t-1} and a low ceiling for human capital accumulation. This means that it will be hard to accumulate human capital which confirms the substitution effect towards increased fertility. When the cost of education, τ, is low, this means that the ceiling is high and so there will be more human capital accumulation.

2.2 Social Planner

The social planner makes decisions based on all households across all periods. The planner also internalizes the complementarities in aggregate human capital, which the household does not see. A cross section of what the planner will be maximizing will be,

$$\begin{aligned} \ldots &+ \beta^{s}\left\{\ln\left[1-\theta-(\phi h_{t-2}^{\varepsilon}+\tau h_{t-1})n_{t-1}\right]w_{t-1}+\ln\left[\theta w_{t-1}n_{t-2}\right]\right\} \\ &+ \beta^{s+1}\left\{\ln\left[1-\theta-(\phi h_{t-1}^{\varepsilon}+\tau h_{t})n_{t}\right]w_{t}+\ln\left[\theta w_{t}n_{t-1}\right]\right\} \\ &+ \beta^{s+2}\left\{\ln\left[1-\theta-(\phi h_{t}^{\varepsilon}+\tau h_{t+1})n_{t+1}\right]w_{t+1}+\ln\left[\theta w_{t+1}n_{t}\right]\right\} \\ &+ \beta^{s+3}\left\{\ln\left[1-\theta-(\phi h_{t+1}^{\varepsilon}+\tau h_{t+2})n_{t+2}\right]w_{t+2}+\ln\left[\theta w_{t+2}n_{t+1}\right]\right\}+\ldots \end{aligned}$$

Because the planner recognizes the role of aggregate human capital in wages, $H_{t-1}^{\delta} = \left(n_{t-1}h_{t-1}n_{t-2}\right)^{\delta}$ is substituted in,

$$\begin{aligned} \ldots &+ \beta^{s}\left\{\ln\left[1-\theta-(\phi h_{t-2}^{\varepsilon}+\tau h_{t-1})n_{t-1}\right]n_{t-2}^{\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}+\ln\left[\theta n_{t-2}^{1+\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}\right]\right\} \\ &+ \beta^{s+1}\left\{\ln\left[1-\theta-(\phi h_{t-1}^{\varepsilon}+\tau h_{t})n_{t}\right]n_{t-1}^{\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}+\ln\left[\theta n_{t-1}^{1+\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}\right]\right\} \\ &+ \beta^{s+2}\left\{\ln\left[1-\theta-(\phi h_{t}^{\varepsilon}+\tau h_{t+1})n_{t+1}\right]n_{t}^{\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}+\ln\left[\theta n_{t}^{1+\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}\right]\right\} \\ &+ \beta^{s+3}\left\{\ln\left[1-\theta-(\phi h_{t+1}^{\varepsilon}+\tau h_{t+2})n_{t+2}\right]n_{t+1}^{\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}+\ln\left[\theta n_{t+1}^{1+\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}\right]\right\}+\ldots \end{aligned}$$

After solving the first order condition for fertility at period *t*,

$$n_t = \frac{\beta(1+2\delta(1+\beta))}{(1+\beta(1+2\delta(1+\beta)))}\left[\frac{(1-\theta)}{(\phi h_{t-1}^{\varepsilon}+\tau h_t)}\right] \tag{12}$$

As can be seen above, fertility here still depends on the same things as equation (7) except for the added δ term, the parameter associated with the aggregate human capital specification. In fact, if the parameter δ is zero, taking out the aggregate human capital's effect on the social planner's decision, equation (11) becomes the solution to the household's first order condition for fertility, equation (7). The first order condition for human capital at period *t*, is more difficult to solve cleanly. After substituting in equation (11), the simplest form that I can reduce it to is,

$$\frac{\tau h_t(1+2\delta)}{\phi h_{t-1}^{\varepsilon}+\tau h_t} = \left[2(\delta+\gamma) - \frac{\beta\phi\varepsilon h_t^{\varepsilon}(1+2\delta)}{\phi h_t^{\varepsilon}+\tau h_{t+1}}\right]$$

Since it is not possible to solve it without specifying some of the parameters, I decided to move straight to finding the steady state for human capital from the social planner's perspective. When $h_{t+1} = h_t = h_{t-1}$,

$$h^*_p = \left[\frac{\phi}{\tau}\frac{2(\gamma+\delta) - \varepsilon\beta(1+2\delta(1+\beta))}{(1+2\delta(1+\beta)) - 2\gamma}\right]^{\frac{1}{1-\varepsilon}} \tag{13}$$

From this, it is also possible to see how the pre-existing relationships from the human capital steady state from the household's perspective carried over. The negative relationship with the cost of education and the positive relationship with the cost of having children still hold. For this steady state to be positive and not infinity, the returns to education, γ, has to obey the inequality,

$$\gamma - \delta(1+\beta) < 0.5 \tag{14}$$

To determine whether households suboptimize in their investment decisions, the steady state human capital from the social planner's standpoint, h^*_p, has to be larger than the steady state human capital from the household's standpoint h^*_i. This then makes the case for an education subsidy. Confirming the suboptimization, as seen later, will require actually estimating some of the parameters. A simple comparison, however, can be made by taking away effects in h^*_p that are not found in the household steady state. Since the social planner's steady state human capital contains the parameter behind the opportunity cost of having children, ε, and the inter-temporal elasticity of substitution, β, it means that more than the positive externalities from aggregate human capital, δ, is effecting the social planner's decision. To prove this, the returns to aggregate human capital effect is removed, i.e. $\delta = 0$,

$$h^{**}_p = \left[\frac{\phi}{\tau}\frac{2\gamma - \varepsilon\beta}{1-2\gamma}\right]^{\frac{1}{1-\varepsilon}} \tag{15}$$

The presence of the opportunity cost term, ε, shows that the social planner is also taking into account the fertility cost increase imposed by the parents on their children for raising their grandchildren in the next generation. The human capital bought for each child translates into an opportunity cost for them in having children the next period. However, parents who are investing in human capital for their children are only concerned with maximizing the utility of their consumption during their retirement. The parents are not taking into account the fertility cost they will impose on their children during their children's productive period. This cost will cause the parents to choose a higher human capital investment rate. The assumption of this inequality will mean that from the social planner's perspective, households are under-investing in their children's human capital rather than over-investing. If this effect is also removed, i.e. $\varepsilon = 0$,

$$h^{***}_p = \left[\frac{\phi}{\tau}\frac{2\gamma}{1-2\gamma}\right]^{\frac{1}{1-\varepsilon}} \tag{16}$$

From this, the similarity with equation (10), the household's steady state human capital can be seen. After comparing equation (16) to equation (10), it is evident that without the aggregate effects of complementarities in human capital or the incurred cost on the grandchildren's generation, the social planner's steady state is still higher than the household's, as seen below,

$$\left[\frac{\phi}{\tau}\frac{2\gamma}{1-2\gamma}\right]^{\frac{1}{1-\varepsilon}} > \left[\frac{\phi}{\tau}\frac{\gamma}{1-\gamma}\right]^{\frac{1}{1-\varepsilon}}$$
$$2\gamma > \gamma$$

Moving to the steady state fertility decision from the social planner's standpoint,

$$n^*_p = \frac{\beta(1-\theta)(1+2\delta(1+\beta))}{(1+\beta(1+2\delta(1+\beta)))}\left(\frac{((1+2\delta(1+\beta))-2\gamma)}{\phi}\right)^{\frac{1}{1-\varepsilon}}\left[\frac{\tau}{2(\gamma+\delta)-\varepsilon\beta(1+2\delta(1+\beta))}\right]^{\frac{\varepsilon}{1-\varepsilon}}$$

The basic relationships here hold. There is a negative relationship with the cost of fertility but a positive relationship with rising education cost. The aggregate and opportunity cost effects are harder to determine just by looking at the equation. However, taking these effects out, it is extremely close in form to (11),

$$n^{***}_p = \frac{\beta(1-\theta)}{(1+\beta)}\left(\frac{(1-2\gamma)}{\phi}\right)^{\frac{1}{1-\varepsilon}}\left(\frac{\tau}{2\gamma}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

Since all the equations here are relatively complex, further work on the parametric relationships within them and comparisons between them will require specifying the parameter values. Table 1 shows the parameters and their estimated values.

Parameter	Description	Value	Note
γ	Returns to individual human capital	0.02	Effect of total education on real GDP output in 1980 (Lau, Jamison and Louat, 1991)
τ	Cost of education	0.05	Completed secondary education of low and low middle income countries in 1980 as proxy (Barro and Lee, 2000)
θ	Cost of parental support	0.04	Ratio of population over 65 over total population of low and low middle income countries in 1980 as proxy (Health, Nutrition and Population Database)
β	Inter-temporal elasticity of substitution	0.03	Average annual estimate, 0.9 taken to the 30th power to represent the span of a generation. $(0.9)^{30}$
ε	Returns to opportunity cost of fertility	0.01	Assumed value relative to γ to ensure continual investment in education
ϕ	Cost of childcare	0.1	Assumed value relative to τ in that childcare is a more consistent cost
δ	Returns to aggregate human capital	0.01	Assumed value relative to γ in that there are more individual returns to human capital compared to the aggregate

Table 1 Parameter estimates

If h^*_p is to be larger than h^*_i, the following inequality will have to apply,

$$\left[\frac{\phi}{\tau}\frac{2(\gamma+\delta)-\varepsilon\beta(1+2\delta(1+\beta))}{(1+2\delta(1+\beta))-2\gamma}\right]^{\frac{1}{1-\varepsilon}} > \left[\frac{\phi}{\tau}\frac{\gamma}{(1-\gamma)}\right]^{\frac{1}{1-\varepsilon}}$$

$$2(\gamma+\delta-\gamma\delta)\delta > (\gamma+(1-\gamma)\varepsilon\beta)(1+2\delta(1+\beta))$$

This inequality's feasibility can be tested in Mathematica by varying two parameters at a time, holding the others at their estimated values. Mathematica generates plots that are shaded for the regions where the inequality holds across the range of both parameters. This inequality holds for most of the parameters when they are varied from zero to one[5]. An example is Figure 1 below, where the inequality holds for the entire ranges of both parameters δ and ε. The initial

[5] The parameters are varied only from zero to one because they are positive fraction as discussed earlier.

estimated values from Table 1 for δ and ε are shown in the plot. This shows the wide range of values for δ and ε where the inequality will still apply.

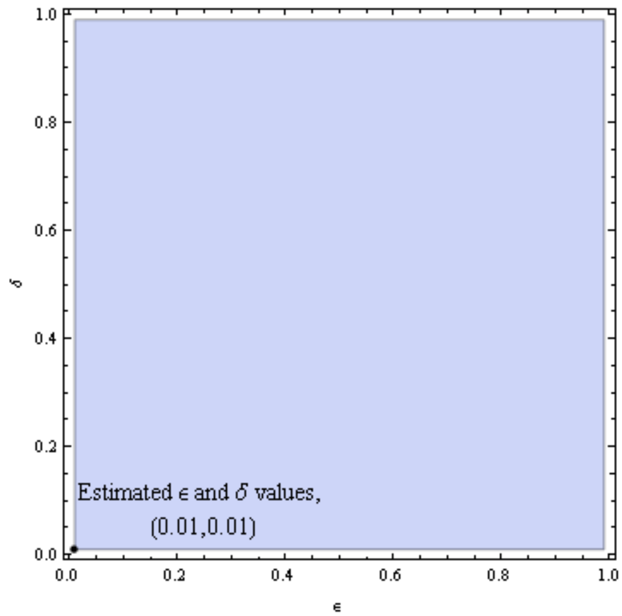

Figure 1 Social planner's higher steady state (SS) HC: Inequality feasibility plot for δ versus ε

Figure 2 shows the feasibility plot for individual and aggregate human capital returns. From the inequality in (14), γ should be less than 0.5. As can be seen, the inequality applies for the entire range of both parameters. Again, the initial estimated values for both parameters are shown. The last plot is included in the appendix. These plots confirm that households invest less in human capital compared to the social planner at steady state.

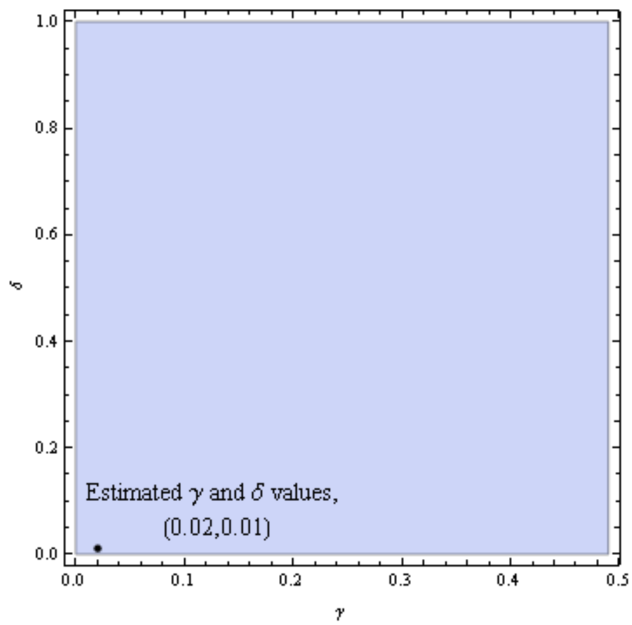

Figure 2 Social planner's higher SS HC: Inequality feasibility plot for δ versus γ

As it has been established that there is a need for an education subsidy due to suboptimal investments from the part of the household, it should also be shown that they are having more children than what the social planner would prefer. If n^*_p is to be smaller than n^*_i, the following inequality will have to apply,

$$\left(\frac{1-\gamma}{((1+2\delta(1+\beta))-2\gamma)}\right)^{\frac{1}{1-\varepsilon}}(1+\beta(1+2\delta(1+\beta)))>(1+\beta)(1+2\delta(1+\beta))\left[\frac{\gamma}{2(\gamma+\delta)-\varepsilon\beta(1+2\delta(1+\beta))}\right]^{\frac{\varepsilon}{1-\varepsilon}}$$

Using estimated parameters, the same inequality plotting can be done with Mathematica. The majority of shaded areas confirm that the households are over-investing in fertility compared to the social planner's perspective. Again, the estimated values for from Table 1 for both varied parameters are shown below in each plot. The other feasibility plots are included in the appendix.

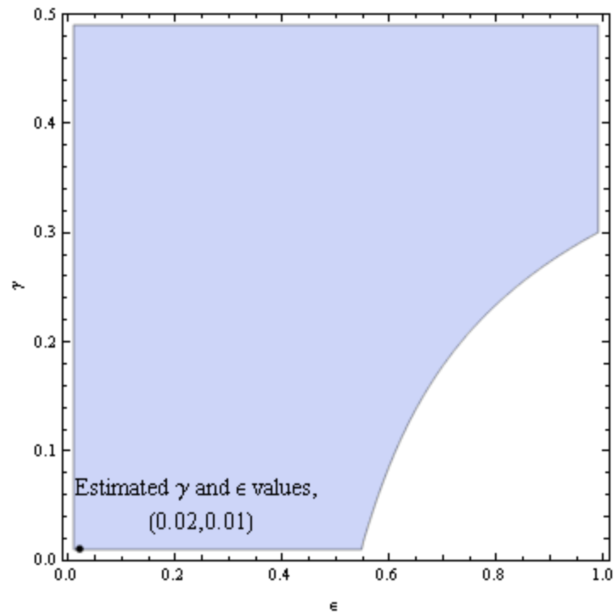

Figure 3 Social Planner's lower SS Fertility: Inequality feasibility plot for γ versus ε

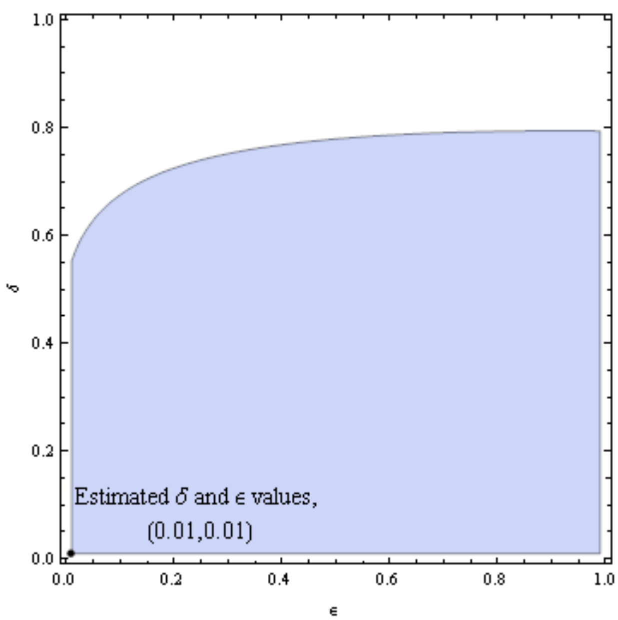

Figure 4 Social Planner's lower SS Fertility: Inequality feasibility plot for δ versus ε

Since the majority of the regions in the plots above are shaded, the suboptimality of the households' investment decisions in children can be established[6]. As a result, a policy measure to

[6] The further prove this, the steady states for fertility and human capital for both household and social planner are plotted on graphs with the estimated parameter values. These graphs are included in the appendix.

close these gaps can be introduced, be it a subsidy on human capital investment or tax on fertility. In this paper, I choose to look at an education subsidy instead of a fertility tax because education subsidies are the most popular policy instrument used by governments to increase human capital investments and subsequently reduce fertility. There are a number of countries like China and Singapore who have taxed having children in order to lower fertility rates, but these policies are not that common and it can be assumed to be a permanent positive shock to fertility cost in the model.

To calculate how much of a subsidy would be necessary, the cost of education, τ, for the household is broken up into the actual cost of education and a subsidy, $(\tau - s)$. Since the household is investing less than the desired amount, the cost of education for them should be higher than what it actually is with the complementarities internalized. The subsidy will lower that cost. The planner's level of investment will just be the actual cost of education. Equating both the steady states from the household and the social planner, the optimal subsidy is as follows,

$$\tilde{s} = \frac{\tau\left[\gamma + (1-\gamma)(2\delta - \varepsilon\beta) - (\varepsilon\beta(1-\gamma) + \gamma)(2\delta(1+\beta))\right]}{(1-\gamma)\left[2(\gamma+\delta) - \varepsilon\beta(1+2\delta(1+\beta))\right]} \tag{17}$$

For the subsidy to not be negative or infinity,

$$2(\gamma + \delta) > \beta\varepsilon(1 + 2\delta(1+\beta)) \tag{18}$$

The positive fraction γ has to obey inequality (14). After substituting (14) into (18), the inequality is now

$$\varepsilon\beta - 2\delta < 1 + 2\delta(1+\beta)(1-\varepsilon\beta)$$

Since parameters β, δ, and ε are all positive fractions , the left hand side of the inequality will always be smaller than the right hand side. This confirms that the subsidy does not go to infinity.

As can be seen from equation (17), there is a positive relationship between the amount of subsidy and the education cost. As education cost increases, so will the subsidy. This can be shown by differentiating τ with respects to $\tilde{s}$ and getting a positive result.

$$\frac{d\tilde{s}}{d\tau} = \frac{[\gamma + (1-\gamma)(2\delta - \varepsilon\beta) - (\varepsilon\beta(1-\gamma) + \gamma)(2\delta(1+\beta))]}{(1-\gamma)[2(\gamma+\delta) - \varepsilon\beta(1 + 2\delta(1+\beta))]}$$

By contour plotting the effect of τ on $\tilde{s}$ in Mathematica, the positive relationship can be confirmed.

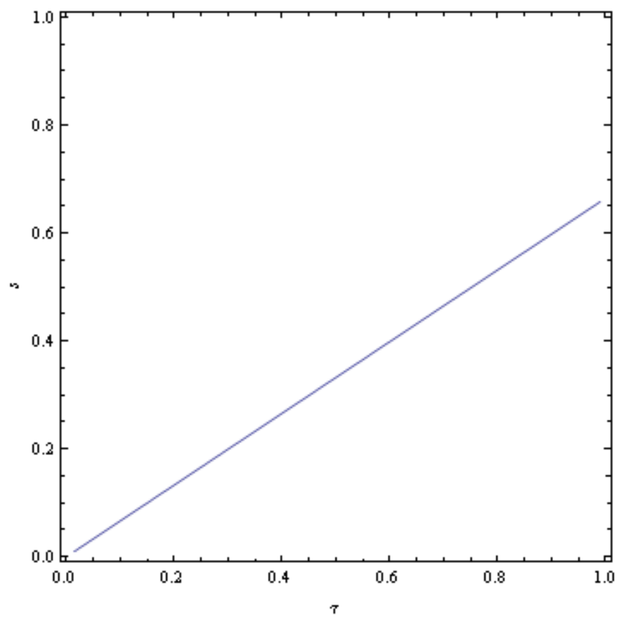

Figure 5 Contour plot of education subsidy *s* versus aggregate returns τ

The rest of the parameter relationship within the optimal subsidy condition is less obvious at first glance and so they are plotted using Mathematica with the earlier estimates. Below is the relationship between the aggregate human capital effects and the subsidy,

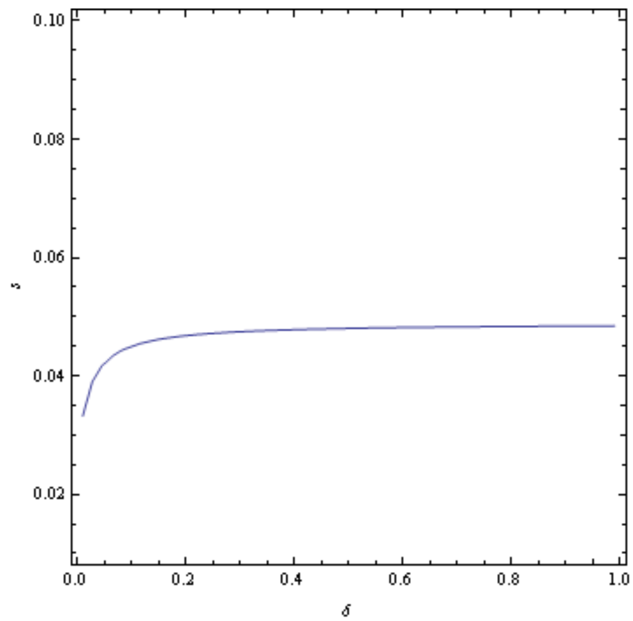

Figure 6 Contour plot of education subsidy *s* versus aggregate returns δ

As the positive externalities from human capital increases, so will the optimal education subsidy. This confirms the initial intuition of the social planner taking into account the positive externalities of education on society and thereby encouraging households to invest in more human capital by increasing education subsidy. However, it is not a linearly increasing relationship. The subsidy increases at a much slower rate at higher levels of aggregate returns compared to lower levels. This convergence confirms the theory of a steady state that countries will move towards.

Figure 7 shows the relationship between the power of opportunity cost of having children and subsidy. The negative relationship between the power of fertility's opportunity cost and subsidy is clearly shown here.

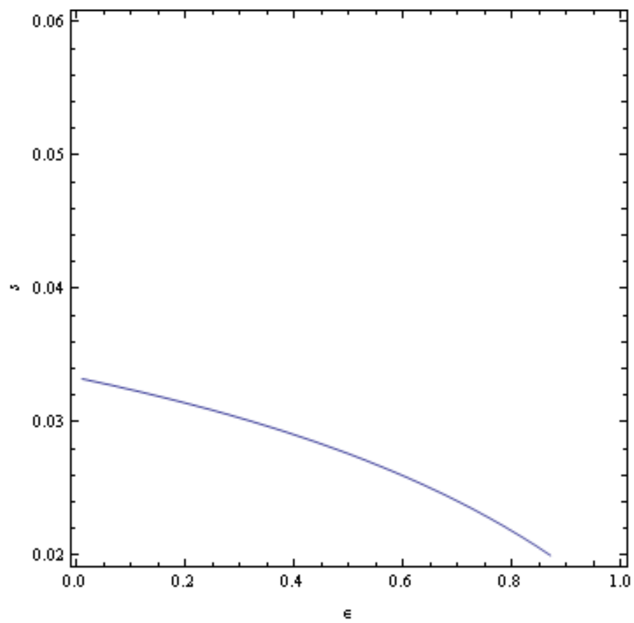

Figure 7 Contour plot of education subsidy *s* versus power of opportunity cost of fertility ε

Figure 8 shows the relationship between the returns to individual human capital and subsidy.

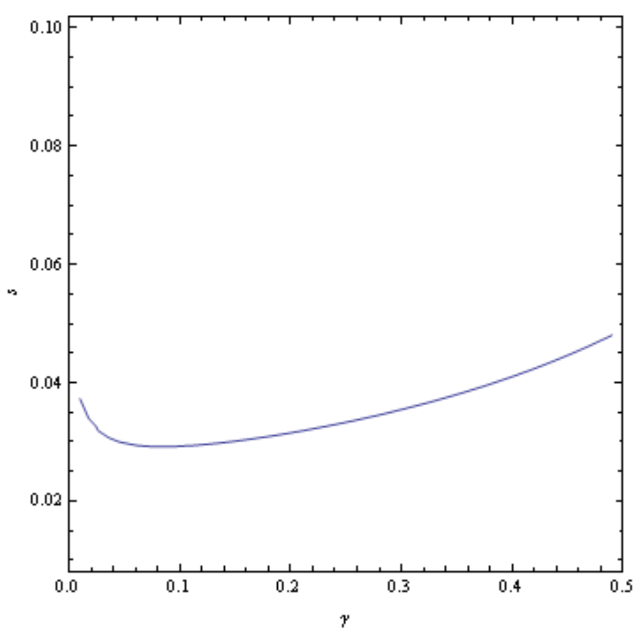

Figure 8 Contour plot of education subsidy *s* versus individual human capital return γ

When the individual human capital returns are extremely low, subsidy decreases initially. This means that at very low values of individual returns, there are not enough positive returns for more education subsidies to be implemented in the economy. However, the subsidy starts to increase exponentially after the γ value of about 0.06, indicating that the positive effect of the individual human capital returns dominates after that.

The optimal subsidy condition here is introduced as an exogenous variable into the model. This is not unrealistic in that foreign aid to a country targeted at encouraging its education investment will have a similar effect on the decisions of its households. Taking it a step further, though, the next section explores the model when the economy has to finance this subsidy on its own.

3 Domestic Policy Implications

3.1 The Decentralized Economy

Now that households have been shown to have suboptimal levels of investments and an optimal subsidy has been established, it is important to document how this policy instrument can be internalized to help channel the households towards the ideal level of investment. Since government expenditure has to equal government revenue, subsidies given out to households has to be financed by the same amount in taxes. The tax will take the form of a payroll tax and will only affect the productive population at each period since they will be the ones earning wages and acting on the subsidy.

Payroll tax takes a portion out of the productive individuals' incomes. This decreases their spending capacity for everything else. However, the education subsidy returns this amount to the household, specifically as a discount in education cost. Since the households are not involved in the tax policy decisions, they take the tax levy, *T,* and the education subsidy value, *s*, as exogenous. The new budget constraint at period *t* will look as follows,

$$c_t = (1-T)w_t\left(1-\theta-(\phi h_{t-1}^{\varepsilon}+(\tau-s)h_t)n_t\right) \tag{17}$$

The payroll tax that is paid, *T*, is a positive fraction that gets taken out of the household's wage. The cost of education, τ, still remains but added on now is, *s*, the education subsidy.

In the retired period, the constraint is

$$c_t = (1-T)w_{t+1}\theta n_t \tag{18}$$

The first order condition for fertility solved for n_t is

$$n_t = \frac{\beta}{(1+\beta)}\frac{(1-\theta)}{\left(\phi h_{t-1}^{\varepsilon}+(\tau-s)h_t\right)}$$

This is very similar to the first order condition for the household before the policy implications and the same relationships within the parameters still apply.

The first order condition for human capital solved for n_t is

$$n_t = \frac{\beta\gamma(1-\theta)}{(1+\beta\gamma)(\tau-s)h_t+\beta\gamma\phi h_{t-1}^{\varepsilon}}$$

Having equated both solved first order conditions, the optimal conditions are,

$$\tilde{\tilde{h}}_t = \frac{\gamma}{1-\gamma}\frac{\phi}{(\tau-s)}h_{t-1}^{\varepsilon} \tag{19}$$

$$\tilde{\tilde{n}}_t = \frac{\beta}{(1+\beta)}\frac{(1-\theta)(1-\gamma)}{\phi h_{t-1}^{\varepsilon}} \tag{20}$$

The relationships within the optimal conditions with payroll tax are exactly the same as before without the addition of the subsidy. With the optimal condition for $\tilde{\tilde{h}}_t$, the subsidy has a positive relationship.

The resultant steady state will be,

$$h^*_{payroll} = \left[\frac{\gamma}{1-\gamma}\frac{\phi}{(\tau - s)}\right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

Everything is the same except for the addition of the subsidy term which will have a positive impact on the steady state of human capital accumulation.

The steady state for fertility with the payroll tax is

$$n^*_{payroll} = \frac{\beta(1-\theta)}{(1+\beta)}\left(\frac{(1-\gamma)}{\phi}\right)^{\frac{1}{1-\varepsilon}}\left(\frac{(\tau - s)}{\gamma}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{22}$$

As can be seen, the added subsidy term has a negative impact on steady state fertility of the household with a payroll tax. The subsidy will motivate more investment in human capital versus fertility. Since the payroll tax is exogenous, the household has to take it as given, pay it, and work with whatever income is leftover after tax. Since they construct their decisions post-tax, the tax term T is not included in both steady states.

3.2 The Social Planner

For the social planner, the payroll tax is internalized. Since government spending has to equal government revenue, the amount taxed has to equal to the amount received.

$$Tw_t n_{t-1} = sn_t h_t n_{t-1}$$
$$T = \frac{sn_t h_t}{w_t} \tag{23}$$

The productive population in the economy will be taxed to finance this subsidy because they are earning the wages. The total payroll tax incurred on all productive citizens is the product of the value of the tax, a positive fraction, the wages of individuals, and the number of productive individuals born the period before, $Tw_t n_{t-1}$. This amount translates into the subsidy, *s*, for the human capital of each child, $n_t h_t$, of all the households, n_{t-1}.

Since the social planner internalizes this tax, the consumption pattern across time now looks slightly different,

$$
\begin{aligned}
\ldots &+ \beta^{s}\left\{\ln\left[\left(1-\frac{sn_{t-1}h_{t-1}}{n_{t-2}^{\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}}\right)n_{t-2}^{\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}\left(1-\theta-(\phi h_{t-2}^{\varepsilon}+(\tau-s)h_{t-1})n_{t-1}\right)\right]+\ln\left[\left(1-\frac{sn_{t-1}h_{t-1}}{n_{t-2}^{\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}}\right)n_{t-2}^{1+\delta}h_{t-2}^{\delta+\gamma}n_{t-3}^{\delta}\theta\right]\right\} \\
&+ \beta^{s+1}\left\{\ln\left[\left(1-\frac{sn_{t}h_{t}}{n_{t-1}^{\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}}\right)n_{t-1}^{\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}\left(1-\theta-(\phi h_{t-1}^{\varepsilon}+(\tau-s)h_{t})n_{t}\right)\right]+\ln\left[\left(1-\frac{sn_{t}h_{t}}{n_{t-1}^{\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}}\right)n_{t-1}^{1+\delta}h_{t-1}^{\delta+\gamma}n_{t-2}^{\delta}\theta\right]\right\} \\
&+ \beta^{s+2}\left\{\ln\left[\left(1-\frac{sn_{t+1}h_{t+1}}{n_{t}^{\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}}\right)n_{t}^{\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}\left(1-\theta-(\phi h_{t}^{\varepsilon}+(\tau-s)h_{t+1})n_{t+1}\right)\right]+\ln\left[\left(1-\frac{sn_{t+1}h_{t+1}}{n_{t}^{\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}}\right)n_{t}^{1+\delta}h_{t}^{\delta+\gamma}n_{t-1}^{\delta}\theta\right]\right\} \\
&+ \beta^{s+3}\left\{\ln\left[\left(1-\frac{sn_{t+1}h_{t+1}}{n_{t+1}^{\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}}\right)n_{t+1}^{\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}\left(1-\theta-(\phi h_{t+1}^{\varepsilon}+\tau h_{t+2})n_{t+2}\right)\right]+\ln\left[\left(1-\frac{sn_{t+1}h_{t+1}}{n_{t+1}^{\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}}\right)n_{t+1}^{1+\delta}h_{t+1}^{\delta+\gamma}n_{t}^{\delta}\theta\right]\right\}+\ldots
\end{aligned}
$$

Say at period *t,* the optimal consumption function of the retired population is c_t^R* and c_t^P* is the function for the productive population,

$$c_t^P* = (1-T)w_t\left(1-\theta-\phi h_{t-1}^{\varepsilon}n_t-(\tau-s)h_t n_t\right) \tag{24}$$

$$c_t^R* = (1-T)\theta w_t n_{t-1} \tag{25}$$

The optimal levels of human capital and fertility investments for the households are then substituted into c_t^R* and c_t^P* and the social planner's optimization problem across periods is solved by maximizing utility with respect to the subsidy[7].

$$\underset{s}{Max}\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t^P*\right)+u\left(c_t^R*\right)\right]$$

The resultant nonlinear equation is as below,

---

[7] Much appreciation goes to Professor Barbara Craig for teaching me this method.

$$\left[\frac{\beta\gamma(1-\theta)}{(1+\beta)}\right]^{2\delta-1}\left[\frac{\gamma\phi}{(1-\gamma)}\right]^{\frac{\gamma-\delta}{1-\varepsilon}}\left(4\delta+\frac{2(\gamma-\delta)-\varepsilon}{1-\varepsilon}\right)(\tau-s)^{-\left(2\delta-1+\frac{\gamma-\delta}{1-\varepsilon}\right)}=\left(\frac{\varepsilon}{1-\varepsilon}\right)s+2\tau$$

As mentioned before, the social planner in this economy is taking into account two different externalities. The first one is the positive externality that aggregate human capital brings and the second is the added cost that parents imposes on their children in the form of the opportunity cost of more education on fertility. By giving their children human capital in the form of education, parents raise the fertility cost for their children. Since my paper is primarily concerned with the externality from aggregate human capital, I simplify the nonlinear equation above by leaving out the opportunity cost of education effect. With $\varepsilon = 0$, the resultant closed-form solution is as follows,

$$s=\tau-\left[\frac{\tau}{2-\delta-\gamma}\left(\frac{\beta\gamma(1-\theta)}{(1+\beta)}\right)^{1-2\delta}\left(\frac{\gamma\phi}{(1-\gamma)}\right)^{\delta-\gamma}\right]^{\frac{1}{1-\delta-\gamma}} \tag{26}$$

The optimal subsidy from the social planner's perspective will never be more than the per unit cost of education unless the parameters are no longer fractions but are larger than one or negative. For ease of understanding how the parametric relationships work within this optimal subsidy specification, the equation can be re-written below as optimal price of education, which is the cost of education less the subsidy paid out.

$$(\tau-s)=\left[\frac{\tau}{2-\delta-\gamma}\left(\frac{\beta\gamma(1-\theta)}{(1+\beta)}\right)^{1-2\delta}\left(\frac{\gamma\phi}{(1-\gamma)}\right)^{\delta-\gamma}\right]^{\frac{1}{1-\delta-\gamma}} \tag{27}$$

From equation (27), it is easier to see the relationships within. As an example, when the aggregate returns to capital, δ, increases, the social planner should increase the subsidy to internalize the positive effects. This will mean the lowering of the optimal prize of education, $(\tau - s)$. This relationship and the others are reflected in the contour plots from Mathematica as follows[8].

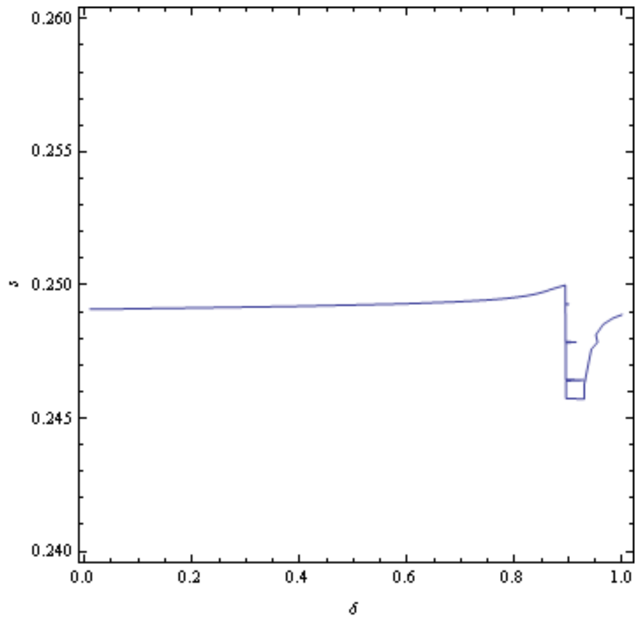

Figure 9 Contour plot of education subsidy *s* versus aggregate returns δ after tax internalization

Figure 9 shows the increase in subsidy allocation as aggregate returns to human capital increases. The sharp drop at $\delta = 0.9$ happens because the estimated parameter for γ is 0.1. When $\delta + \gamma = 1$, the power of the entire benefits term in (26) or the right hand side term in (27) goes to infinity, resulting in the steep drop[9].

[8] The parameter estimates for these contour plots are all multiplied by five to make their effects more visible in the plots.
[9] When $\delta + \gamma = 1$, the power of the entire benefits term in (26) or the entire right hand side term in (27) becomes $\frac{1}{1-\delta-\gamma} = \frac{1}{0} = \infty$. This causes the entire term to increase to infinity.

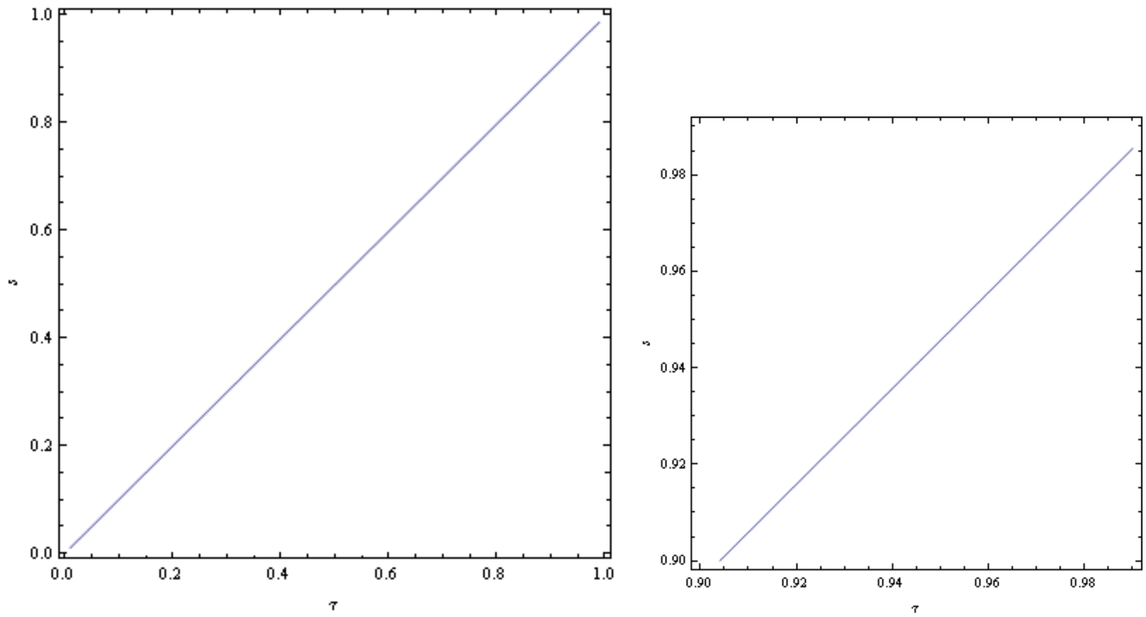

Figure 10 Contour plot of education subsidy *s* versus education costs τ after tax internalization

At first glance, the per unit education cost seems to increase one for one with the education subsidy. However, when the plot is enlarged for very high levels of education cost, the optimal education subsidy is shown to increase at a much slower rate as seen in the smaller plot on the right. Diminishing returns to higher levels of education costs applies with the tapering off of the subsidy allocation.

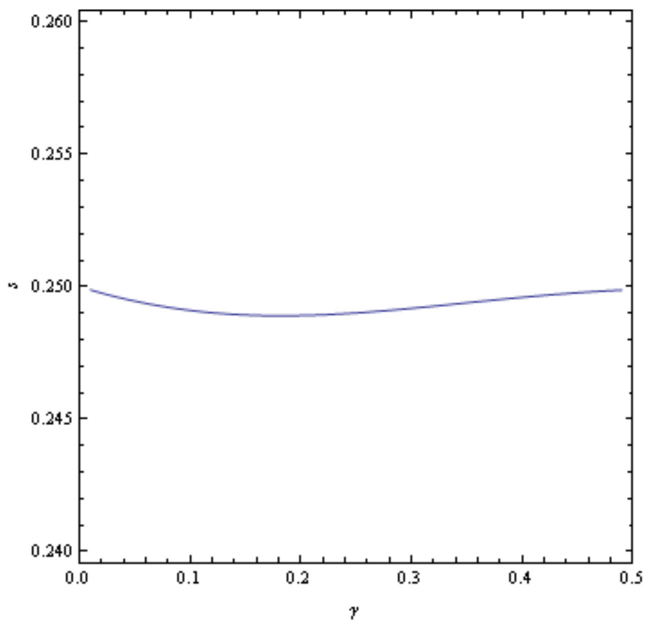

Figure 11 Contour plot of education subsidy *s* versus individual returns γ after tax internalization

The threshold of γ that was seen in Figure 8 is confirmed here. The subsidy initially decreases with low levels of individual returns but then increases after that[10].

A comparison between the optimal education subsidy as a form of foreign aid or when funded by the economy's tax revenues confirms a number of earlier intuitions. The cost of education results in an increase in the optimal education subsidy in both instances. This makes intuitive sense because when education costs more the subsidy should increase accordingly holding everything else constant. Similarly, higher aggregate returns to human capital will result in higher education subsidy allocations. This is because the social planner will have more human capital investment benefits to internalize compared to the households. This positive relationship is confirmed in both specifications as well. The individual returns to human capital have a more dynamic relationship with education subsidy. A threshold of γ is confirmed in both

[10] There is an additional contour plot of the relationship between fertility costs and education subsidy which is included in the appendix. It is not part of the main body because the first specification of education subsidy did not include the fertility cost parameter.

specifications where below it, education subsidy will decrease. However, beyond this value, education subsidy monotonically increases with higher individual returns to human capital. Due to the omission of the opportunity cost of education externality, I do not have a contour plot documenting the relationship between the returns to the parents' opportunity cost and the education subsidy in the second specification. However, if there is a way to simplify the equation and find that relationship, it should be no different from when the subsidy is allocated exogenously.

The next section broadly tests the theoretical model for real world implications.

4 Empirical Implications

4.1 Overview of data

Since education subsidies have been employed by many countries over the years, I have decided to test the effectiveness of this policy measure. My hypothesis is that countries, particularly from the lower income spectrum, are implementing relatively larger subsidies compared to developed countries because they realize that households suboptimize in their human capital investments. These subsidies, if effective, should cause a relatively larger increase in education levels and a relatively bigger decrease in fertility rates in developing countries. I will be presenting only my findings for secondary education here since for many developing countries the number of years for compulsory education includes primary education but doesn't cover secondary education entirely, if at all. Therefore secondary education levels will be responsive to the effects of a subsidy more so than primary education[11].

[11] My findings on primary education have been included in the appendix.

I will first use the difference-in-differences method to test the effectiveness of the subsidies in terms of affecting a higher human capital level and a lower fertility rate. After that I run two regressions to test the magnitude of the subsidy's effects on human capital and fertility in the different countries. Also, I include life expectancy data to capture the old-age support motive effect in my model as well as some dummy variables for different income category indicators and the implementation fertility control policies.

Due to data constraints, I use only years 1980 and 2005 for my analysis instead of a time series set. I assume that from 1980, developing countries implement different subsidy measures and so in 2005 there should be a significant difference in human capital levels and fertility rates. Understandably, this decision will mean that I am not able to distinguish the difference between an increase of 10 percent in education levels for a country from 89% to 99% and another from 40% to 50%. Due to their different starting points, 10% from the former country will mean less compared to the latter country in terms of impact. To compensate for this, I differentiate all the countries in my dataset by Low Income, Low Middle Income, Upper Middle Income, and High Income. This means that the starting levels of these countries will be assumed to be different based on the income category they belong to. These countries are ranked according to Gross National Income per capita (in US Dollars) using historical data from the World Bank Development Indicators in 1987[12].

The data for human capital investment comes from the "International Data on Educational Attainment: Updates and Implications" dataset that Barro and Lee put together in 2000. While most studies measure human capital in terms of years of education completed or

[12] 1987 was used because that is the earliest year I could find and the ranks do not change a significant amount in that short time frame.

literacy rates, Barro and Lee pioneered the new method of using international test scores, results from the international adult literacy survey, and estimates of labor-market outcomes to come up with a more effective measure. They call this a measure of "education attained". In addition to that, they take into account the variation in schooling duration across countries while computing their education completed measure, and their measure is relative to the population in the secondary school age group instead of total population. Since their dataset does not span past 2000, I take the most recent year of their data, 1999, and assume it to apply to 2005[13].

Data for fertility rates, subsidies, and life expectancy come from the World Bank Data Resource, specifically the Education Database and the Health Nutrition & Population Database. Data spanning 3 years on either sides of 1980 and 2005 will be used if the observation from the original year is missing. The fertility rate data represents the number of children that would be born to a woman if she were to live to the end of her childbearing years and bear children in accordance with current age-specific fertility rates. The indicator chosen to proxy for the education subsidy is the public expenditure per student on education in secondary school levels as a percentage of Gross Domestic Product (GDP) per capita. The GDP per capita influence in this indicator is removed to take out the effect it has on the subsidy term. . The life expectancy variable indicates the number of years a newborn infant would live if prevailing patterns of mortality at the time of its birth were to stay the same throughout its life.

To get a preliminary sense of the data, a few scatter plots are shown below. The actual regressions will be done in differences of these values from 1980 to 2005 but it is easier to

[13] The gap of six years should not make much of difference because I am looking at a 25 year difference between 1980 and 2005 in total.

compare them now just in terms of levels. A scatter plot showing the difference in secondary education attainment versus difference in fertility rates is included in the appendix

The two graphs below show the relationship between secondary education attained and fertility rates in 1980 and in 2005. The negative relationship is pronounced in both years. In 2005, most UM countries have moved towards the northwest as a result of rising education attainment and falling fertility rates. The LM countries have also experienced similar trends and have moved up to the general area where the UM countries where in 1980. There is a general decrease in fertility rates for all income categories across the years.

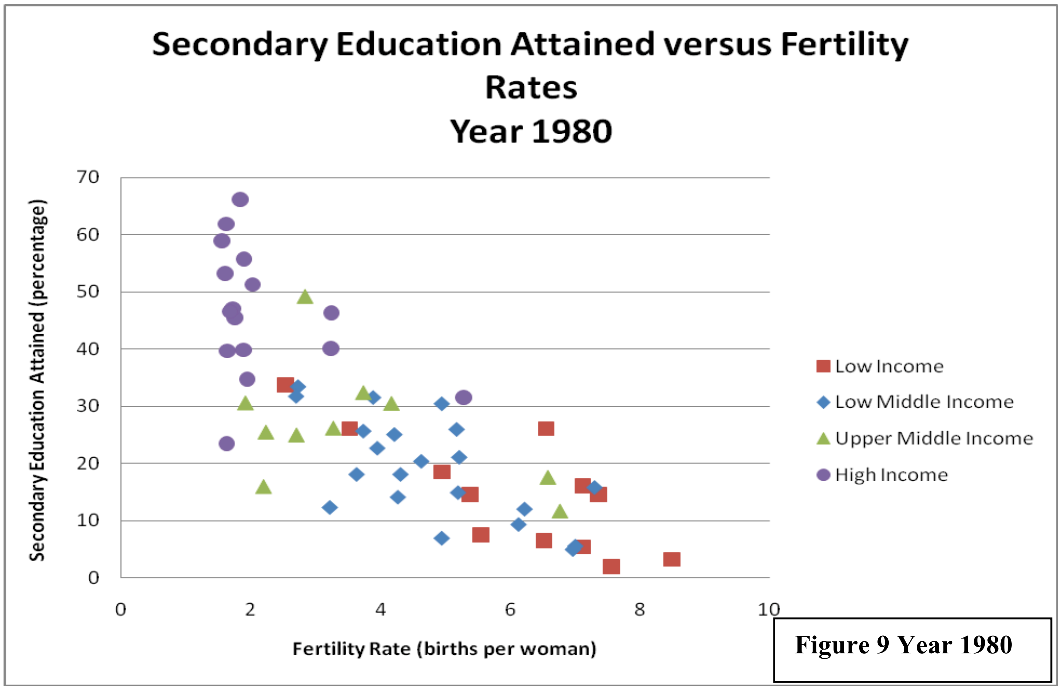

Figure 9 Year 1980

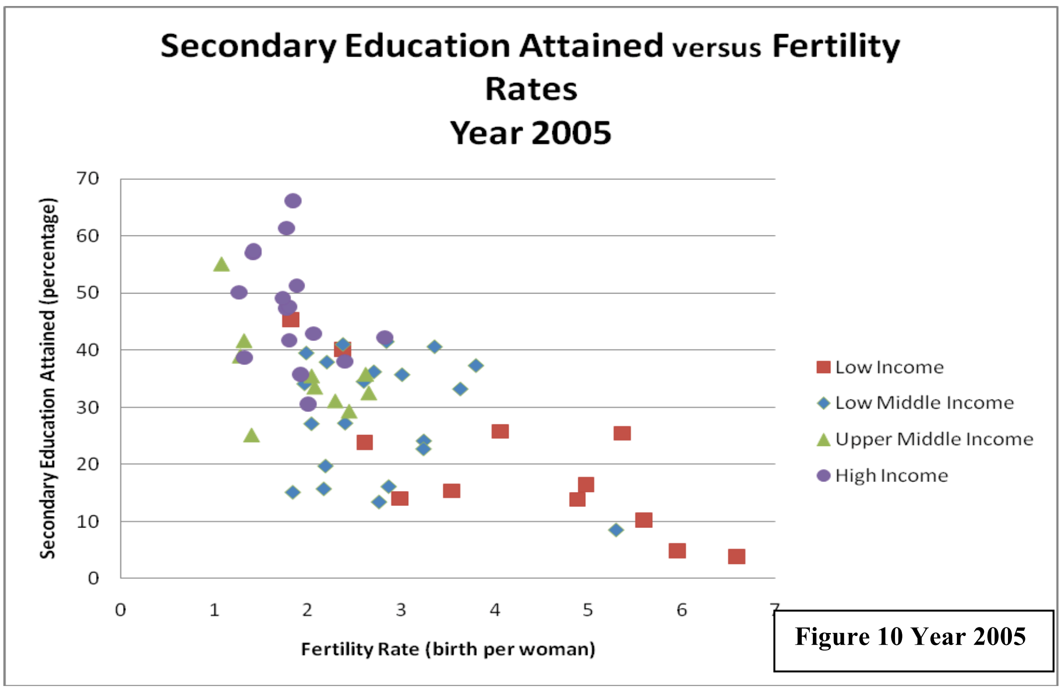

Figure 10 Year 2005

Next, subsidy is introduced on the x-axis to show its effects on fertility rates. The expected negative relationship can be observed as fertility declines and subsidy increases. This southwest trend is led by the H countries and followed by the UM countries in 2005. However the LM countries, and especially the L countries, are slower in responding to the trend.

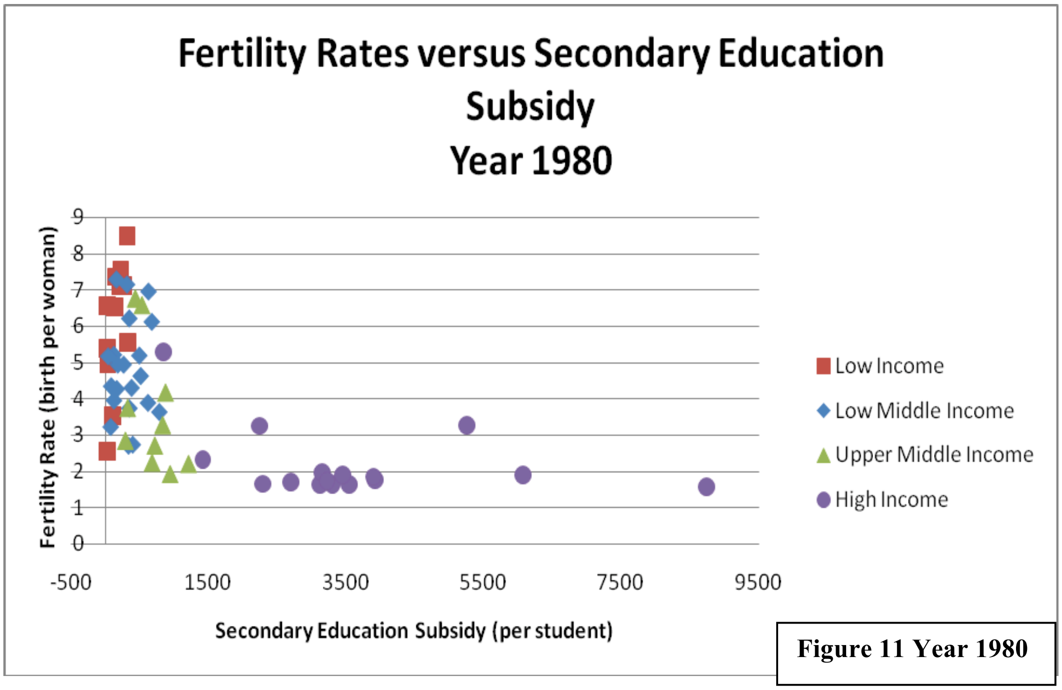

Figure 11 Year 1980

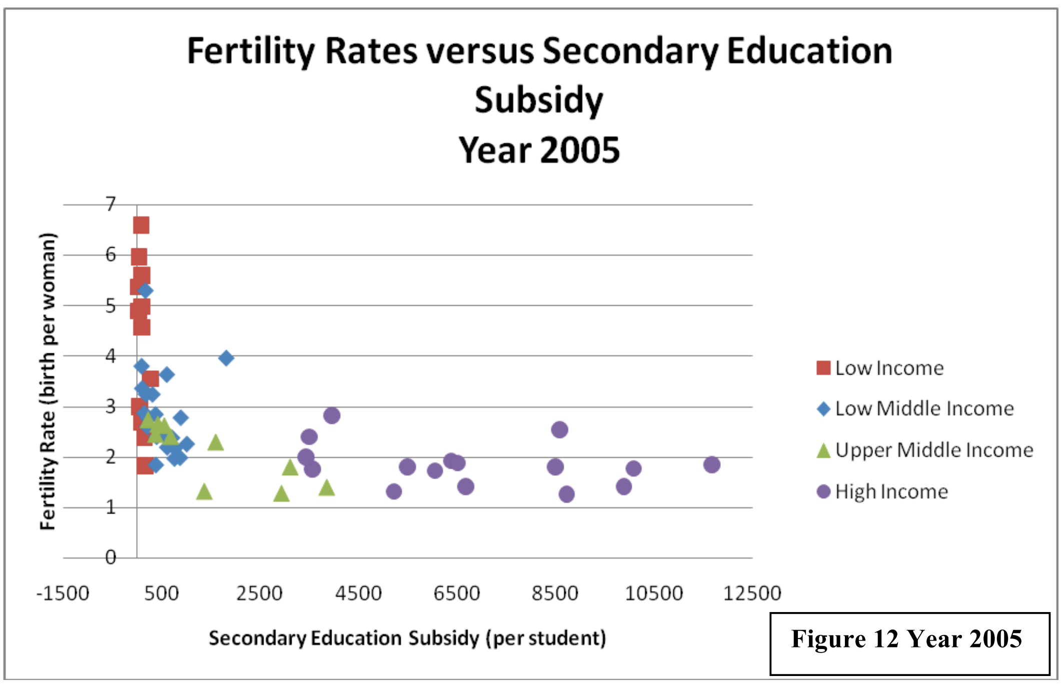

Figure 12 Year 2005

In terms of education attained versus education subsidy. A positive relationship is apparent if both years are compared, again with the H countries leading. The UM countries follows the northeast trend in 2005 but again the LM and L countries lag behind.

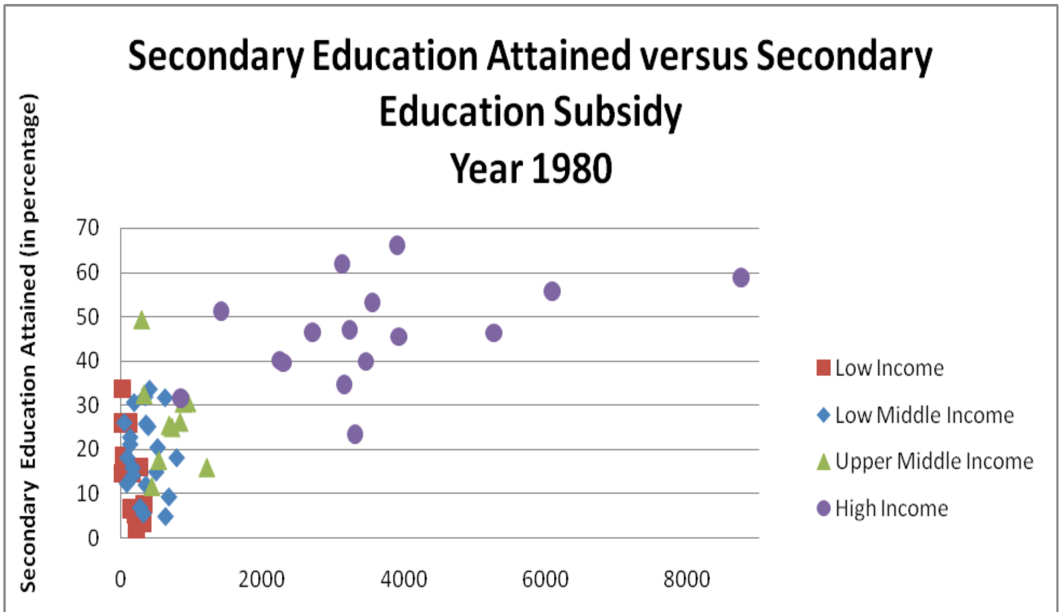

Figure 16 Year 1980

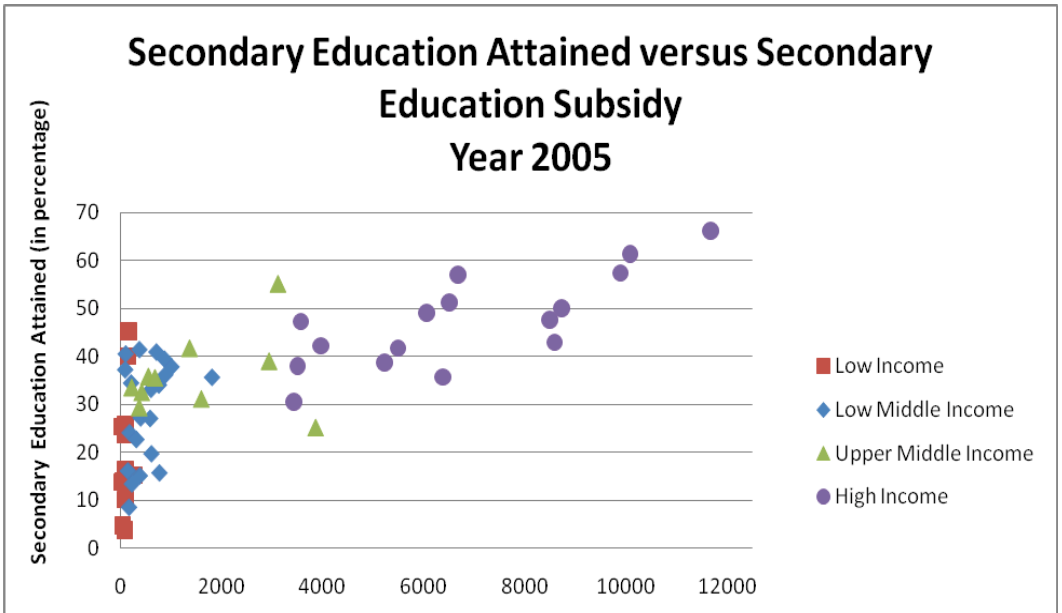

Figure 17 Year 2005

From the scatter plots, it can be deduced that there is a definite negative relationship between fertility and secondary education attainment. The education subsidy's negative effect on fertility and positive effect on secondary education attainment tend to be more pronounced in the

High and Upper Middle income countries with the other lower income countries slowly following suit in 2005.

4.2 Difference-in-difference Analysis

The difference-in-difference analysis involves finding the effect of the policy treatment on a group before the change and after the change, and comparing that with a control group that did not get the same treatment. In this case, the control group will be the developed countries in the High Income category. This is because most of them have 99% literacy rates and high education levels even in 1980. Therefore the rationale is that they have no need for as much education subsidy compared to the countries from other income levels. As a result, the group that has been exposed to the policy change, be it the Low Income (L), Low Middle Income (LM) or Upper Middle Income (UM) group, should have on average a higher secondary education level *increase* or a lower fertility *decrease* compared to the High Income (H) group. By just taking the difference between the groups across the two years, it is assumed that the control group can factor out other changes at the time. This is shown in the expression below,

(Treatment $Group_{2005}$-Treatment $Group_{1980}$) - (Control $Group_{2005}$-Control $Group_{1980}$)[14]

The tables below show the average levels of secondary education attained and completed as well as the fertility rates in 1980 and 2005. The difference-in-difference analysis is also included.

[14] Or (Treatment $Group_{2005}$-Control $Group_{2005}$) - (Treatment $Group_{1980}$-Control $Group_{1980}$)

Income Categories of Countries	No. of Countries	Average Education Completed in 1980 (%)	Average Education Completed in 2005(%)	Average Difference across 25 years	Difference-in-difference Estimate
Low	12	3.116667	4.758333	1.641667	1.066667
Low Middle	21	5.552381	9.395238	3.842857	3.267857
Upper Middle	10	10.82	15.98	5.16	4.585
High	16	20.44375	21.01875	0.575	

Table 2 Difference-in-difference Analysis of Secondary Education Completed

From above, the last column shows the estimates. All treatment groups show a positive increase in completed secondary education relative to the control group. The biggest increase in completed education comes from the UM group while the smallest is from the L group. This is intuitive because UM countries have the incentive and the resources to pursue more aggressive education subsidy policies in order to boost their education levels compared to L countries.

Income Categories of Countries	No. of Countries	Average Education Attained in 1980 (%)	Average Education Attained in 2005(%)	Average Difference across 25 years	Difference-in-difference Estimate
Low	12	14.475	19.88333	5.408333	4.502083
Low Middle	21	19.0619	28.62381	9.561905	8.655655
Upper Middle	10	26.47	35.87	9.4	8.49375
High	16	46.4125	47.31875	0.90625	

Table 3 Difference-in-difference Analysis of Secondary Education Attained

In Table 3, similar effects that are observed from Table 2 are shown here. However, the estimates are greater in magnitude and the LM countries top the UM countries in education attainment by a small percentage. This shows that the treatment groups have a larger positive increase in secondary education attainment relative to the control group.

Income Categories of Countries	No. of Countries	Average Fertility Rate in 1980(births per woman)	Average Fertility Rate in 2005(births per woman)	Average Difference across 25 years	Difference-in-difference Estimate
Low	12	6.058967	4.228367	-1.8306	-1.49611
Low Middle	21	4.775057	2.787363	-1.98769	-1.6532
Upper Middle	10	3.63566	1.91928	-1.71638	-1.38189
High	16	2.158531	1.824042	-0.33449	

Table 4 Difference-in-difference Analysis of Fertility Rate

In terms of fertility rates, all the treatment groups show a decrease in fertility relative to the control group. The LM countries have the biggest drop in fertility followed by the L countries.

It is evident from here that there is a policy effect in the treatment group that is causing a rise in completed secondary education and secondary education attainment as well as a drop in fertility rates. The next step is to introduce the variables in my regressions to test the significance of the subsidy proxy in affecting these changes.

4.3 Regression Analysis

Firstly, to get an accurate education subsidy proxy, the GDP per capita effect has to be removed from the original data taken from the World Bank site. As well, the percentages in the data have to be removed for ease of comparison. If *PuEx* is the public expenditure per student as a percentage of GDP per capita (as given by the World Bank database), and *GDPp* is GDP per capita, then the subsidy term, *sub* can be found by

$$sub = \frac{PuEx * GDPp}{100}$$

The table below shows the breakdown of the average education subsidy in the two years. The L countries have seen a decrease in average education subsidy while the other countries have seen increases. The average H country student has about 24 times the amount of subsidy an average L country student gets in 1980, with the gap widening to about 65 times in 2005. Even the average UM country student gets about 10 times less than a H country student in 1980, but the gap closes to about 5 times in 2005. The H countries have the highest level of average education subsidies as the amount almost doubled from 1980 to 2005. However, the UM countries has the largest relative increase where the 1980 average more than doubled in 2005. The amount of subsidy increase is still dwarfed by the H countries though. The difference in secondary education subsidy is SUB_DIFF.

Income Categories of Countries	No. of Countries	Average Education Subsidy in 1980(per student)	Average Education Subsidy in 2005(per student)	Average Difference across 25 years
Low	12	152.385	103.5521	-48.8329
Low Middle	21	335.076	531.8992	196.8232
Upper Middle	10	684.2215	1503.65	819.4285
High	16	3577.149	6759.365	3182.216

Table 5 Breakdown of education subsidy in 1980 and 2005

The other independent variable for the regression is difference of life expectancy at birth, LEXP_DIFF. The table below shows an increase across all categories for the aged population ratio. The life expectancy in all categories has increased from 1980 to 2005.

Income Categories of Countries	No. of Countries	Life expectancy in 1980 (age)	Life expectancy in 2005 (age)	Average Difference across 25 years
Low	12	52.49174	56.19145	3.699708
Low Middle	21	63.04605	68.0586	5.012551
Upper Middle	10	67.80207	75.0124	7.210334
High	16	74.02969	79.74274	5.713055

Table 6 Breakdown of aged population ratio in 1980 and 2005

The 16 H countries from the previous section have been excluded in the actual regression analysis since I am only concerned with lower income countries in my theoretical model[15]. In the first of two OLS regressions, the dependent variable is the difference in secondary education attained LS_DIFF. The dependent variable in the second regression is the difference in fertility rates FERT_DIFF. Secondary education attained is chosen as the dependent variable in the first regression over secondary education completed because it is a better measure of the human capital stock at the secondary education level across countries.

Lastly, a number of dummy variables will be included. L and UM represent the different income categories of the countries in my dataset. This is so that the individual effects of the subsidy of these categories can be seen on the dependent variable. As well, a dummy variable for fertility control policies FERT_POL is included to show the 23 countries in my sample that have implemented them. The data is taken from the Population Council. The descriptive statistics and measure of correlations for the various independent variables are included in the appendix.

The regression equation for the difference in secondary education attained is

$$LS\_DIFF = \beta_0 + \delta_0 L + \delta_1 UM + \beta_1 FERT\_DIFF + \beta_2 SUB\_DIFF + \beta_3 LEXP\_DIFF$$

Based on the results of my theoretical model, there should be significant increases in the education attainment levels for the income categories L and UM. Life expectancy, LEXP_DIFF, should expect to effect a negative change in the education attainment if resources channeled towards parents takes away from overall income of the children. FERT_DIFF should have a negative correlation with education since they are substitutes in the household's optimization

[15] They were included the in the tables above for comparison purposes.

problem. Lastly, subsidy, SUB_DIFF[16], should have a positive effect on secondary education attainment.

The regression equation for the difference in fertility rates is

$$FERT\_DIFF = \beta_0 + \delta_0 FERT\_POL + \delta_1 L + \delta_2 UM + \beta_1 LS\_DIFF + \beta_2 SUB\_DIFF + \beta_3 LEXP\_DIFF$$

I should expect to see the following relationships based on my model. Fertility control policies FERT_POL should have a negative effect on fertility. Income categories L and UM should be significantly negative towards difference in fertility rates. Life expectancy, LEXP_DIFF, should have a negative effect to fertility rates due to the specification of the variable. The decrease in child mortality and increase in care of parents due to longer life expectancy will decrease overall income of the children. Secondary education attainment LS_DIFF should have a negative correlation with fertility rates because of their substitution effects with each other. Finally, the education subsidy, SUB_DIFF, should have a negative effect on fertility rates since they encourage parents to invest in education instead of fertility.

[16] I did my initial regression with interaction terms between education subsidy and the different income categories. They were all insignificant. Therefore, I have not included them here.

4.4 Results

Below is the first regression result,

Dependent Variable: LS_DIFF
Method: Least Squares
Date: 04/28/09 Time: 10:47
Sample (adjusted): 1 43
Included observations: 43 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	7.254888	2.517085	2.882258	0.0065
L	-4.180017	2.192162	-1.906801	0.0643
UM	0.964594	2.461578	0.391860	0.6974
FERT_DIFF	-2.254130	0.940357	-2.397100	0.0217
SUB_DIFF	0.001251	0.001488	0.840524	0.4060
LEXP_DIFF	-0.516269	0.124589	-4.143777	0.0002

R-squared	0.434645	Mean dependent var	8.365116
Adjusted R-squared	0.358245	S.D. dependent var	7.395748
S.E. of regression	5.924704	Akaike info criterion	6.524926
Sum squared resid	1298.778	Schwarz criterion	6.770675
Log likelihood	-134.2859	Hannan-Quinn criter.	6.615551
F-statistic	5.689110	Durbin-Watson stat	1.916943
Prob(F-statistic)	0.000546		

Table 7 Regression Results for Difference in Secondary Education Attainment

The L dummy variable is negative and significant while the UM variable has an insignificant estimate. This shows that low income countries have a negative effect on the levels of secondary education attained. Next, fertility rates have a highly negative relationship with secondary education attainment, confirming the implications from the theoretical model. Education subsidy has a positive significant impact on secondary education attained also confirming the implications from the theoretical model. Life expectancy has an expected significantly negative relationship with education attained, showing a diverting of funds away from children towards the productive population's parents in their retirement. This again confirms the theoretical model's intuition.

The next set of regression results are as follows,

Dependent Variable: FERT_DIFF
Method: Least Squares
Date: 04/28/09 Time: 10:46
Sample (adjusted): 1 43
Included observations: 43 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.249334	0.399839	-3.124593	0.0035
FERT_POL	-0.499403	0.320339	-1.558981	0.1277
L	0.045583	0.369458	0.123377	0.9025
UM	-0.122063	0.424971	-0.287226	0.7756
LS_DIFF	-0.053021	0.024776	-2.139994	0.0392
SUB_DIFF	0.000456	0.000233	1.962018	0.0575
LEXP_DIFF	-0.017279	0.024096	-0.717066	0.4780

R-squared	0.254943	Mean dependent var	-1.940158
Adjusted R-squared	0.130767	S.D. dependent var	1.014193
S.E. of regression	0.945560	Akaike info criterion	2.873821
Sum squared resid	32.18699	Schwarz criterion	3.160528
Log likelihood	-54.78715	Hannan-Quinn criter.	2.979549
F-statistic	2.053075	Durbin-Watson stat	2.017468
Prob(F-statistic)	0.083601		

Table 8 Regression Results for Difference in Fertility Rates

From these results, the L and UM countries have a insignificant variables. Secondary education attainment is significantly negative with fertility rate, confirming the relationship from the first regression. Education subsidy here however affects fertility positively and significantly, which is not what was expected. Life expectancy has a negative but insignificant effect on fertility.

The strongest relationship that corresponded with my theoretical model is the negative correlation between secondary education attainment and fertility rates. This confirms the derivatives from equations (6) and (7) that are negative for both human capital with respect to fertility and vice versa. The dummy variables for the income categories are mostly insignificant, reiterating the evidence from the scatter plots that the lower income countries are following the lower fertility and higher education trends but at a slower rate. The H countries are the ones leading these trends but they are not part of the regressions. The fertility control policies that were implemented by the 23 countries are not significant in explaining the variability in fertility rates. This might be caused by insufficient information regarding the fertility control policies of other countries.

The life expectancy variable performed as expected in the education regression but not in the fertility one. This is because life expectancy is correlated with fertility rates in that how many children parents plan to have can depend on how many children they think will survive infancy and childhood. This correlation resulted in a lower significance level for life expectancy in the fertility regression. However, the old-age support motive is confirmed in the education regression.

SUB_DIFF, the education subsidy proxy performed as expected in the education regression but not so in the fertility regression. Upon closer analysis of my choice of a proxy for education subsidy and the regression set-up, a number of things became clear. Initial regressions with interaction terms of education subsidy and the different countries' income categories have come up mostly insignificant for both regressions. Therefore they are dropped from the regressions here. From the graphs earlier in the empirical section, it is also clear that most of the positive relationship with secondary education attainment and negative relationship with fertility rates are driven by high income countries. Therefore, I have come to conclude that public expenditure on secondary education per student is not a good proxy to measure education subsidy. Firstly, this proxy only takes into account public expenditure in education and not the private sector investments. Tooley and Dixon (2006) reported on the *de facto* 'privatization' of education due to the poor quality of government-funded schools. Amongst the examples they cited are Uganda, Kenya, and India, which are in the dataset. This report then does four case studies in four cities in Ghana, Nigeria and India to conclude that even though private schools are not part of the solution for satisfying the Millennium Development Goals (MDGs), perhaps that ruling should be revised if privatization of education is the *de facto* solution to educating the future generation in some countries.

Furthermore, many creative strategies have been developed to encourage secondary education in households, particularly for lower income countries. 'Meeting the Challenges of Secondary Education in Latin America and East Asia', a book published by the World Bank in 2006 highlights Latin America and East Asia's efforts in strengthening their secondary education systems. Examples of countries that are also in my dataset include Korea's public-private partnerships to finance its school systems, China's school self-funding strategies including school-run businesses, and Mexico's conditional cash transfer programs for the poor, contingent upon them sending their children to school consistently. Firstly these strategies show that education subsidies are not just publicly funded. In fact, these strategies are developed because many of the governments of poorer countries cannot afford to pay out these subsidies. Secondly, they are highly successful and so is probably the cause for the large positive difference-in-difference estimates for secondary education in the countries they were implemented in.

As well, from the theoretical model the total cost of education is $(\tau - s)$. It is the education cost minus the education subsidy. In the regression I had the education subsidy proxy. However, without a proxy for education cost, it is not possible to know what the net cost is. Therefore a better measure will be the net cost of education after taking out the subsidy term from education cost. Lastly, the regression did not control for extraordinary circumstances. An example of this is the civil war in Rwanda that was happening during 1980 and 2005. As Rwanda's economy is crippled, this resulted in a huge subsidy drop of about 272 dollars per student. The Iran-Iraq war also happened between 1980 and 1988. Their subsidy decreased by about 318 dollars per student. These huge dips in subsidies are caused by external circumstances but since they were included in the regression they heavily skewed the subsidy variable. I chose to not remove them from the dataset because I only had 43 observations to start with.

5 Conclusions

In this paper, I develop a theoretical model to show how positive externalities from aggregate human capital are not internalized by households in their investment decisions. As a result, I show that households over-invest in fertility and under-invest in education for their offspring. By showing that the desired human capital investment levels for the social planner is higher than the household's, I am able to specify an optimal education subsidy that will close the gap between the two parties. Taking it further, I internalize the cost of providing that subsidy in the economy with a payroll tax and was able to detail the resultant subsidy measure. The initial parametric relationships all hold in the new subsidy measure. Empirically, the difference-in-differences analysis shows strong effects of policy implementations resulting in decreased fertility rates as well as increased secondary education attainment in the treatment group compared to the control groups. The OLS regressions show the substitution effect between fertility rates and secondary education attainment from the theoretical model is also confirmed here. Life expectancy as a proxy for the old-age support motive reflects partially the family's added financial expense to care for the elderly. Lastly, the subsidy proxy has proven to be inadequate in capturing all the secondary education level increases and fertility rate decreases for the reasons given above.

There are much opportunity for further work on both the theoretical model and empirical specifications. Firstly, the consumption function in the second period could be constructed to be more realistic. Bequests can be built in, even if minimal, so that parents are not just financial burdens on their children when they are retired. This will make the model more realistic. As well, the lagged tax policy can be introduced so that one generation pays out to another instead of the incumbent measure. This is so that the same generation is not seeing a net change in income and hence the aggregate factors are not affected. It will also make the model more realistic. The fraction paid out to parents, θ, can also be endogenized so the productive population has a choice in deciding how much to pay out to their parents. Lastly, a fertility control measure can be introduced as part of the fertility decisions so that its effect can be studied singularly.

On the empirical section, a more encompassing subsidy measure can be used. It will have to be able to include public expenditure, private investments, and the other funding strategies that were mentioned. One possibility is to use a ranking system. This will ensure that all efforts to increase human capital investment are taken into account. Secondly, more observations can be incorporated to substantiate the findings. The Health, Nutrition, and Population Database as well as the Barro and Lee dataset are quite substantial, so that will be possible if the good subsidy measure has enough observations as well. A more representative variable on the old age support motive should also be found. Rather than just taking the life expectancy, survey data could be used to give a better picture of the aged population's contributions to the household. Also, a measure of the efficiency of different fertility policies, rather than just a dummy variable, will help explain the effects of the fertility regression better.

Finally, an entirely different alternative to testing the theoretical model that initially motivated me to embark on the theoretical model is to look at country-specific data. By focusing on just a few countries, individualized parameters for the respective countries that are dynamic can be estimated and an optimal subsidy level can be determined. This can be compared to their current initiatives in subsidies and their effectiveness can then be measured. This will require more micro-data on these chosen countries but it can be an effective way of testing the validity of my theoretical model's specifications.

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Appendix

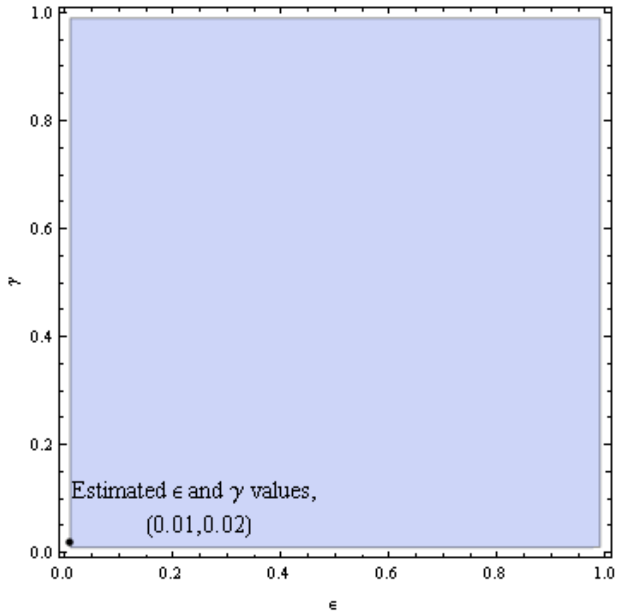

Figure 138 Social planner's higher steady state (SS) HC: Inequality feasibility plot for γ versus ε

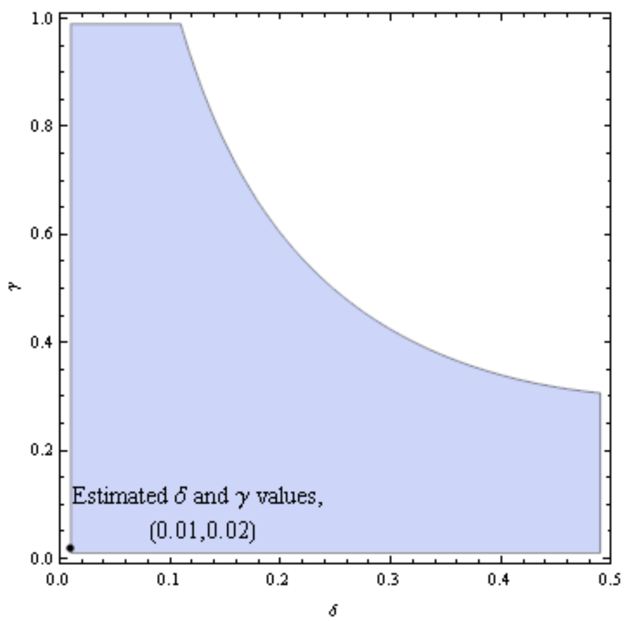

Figure 19 Social planner's lower steady state (SS) Fertility: Inequality feasibility plot for γ versus δ

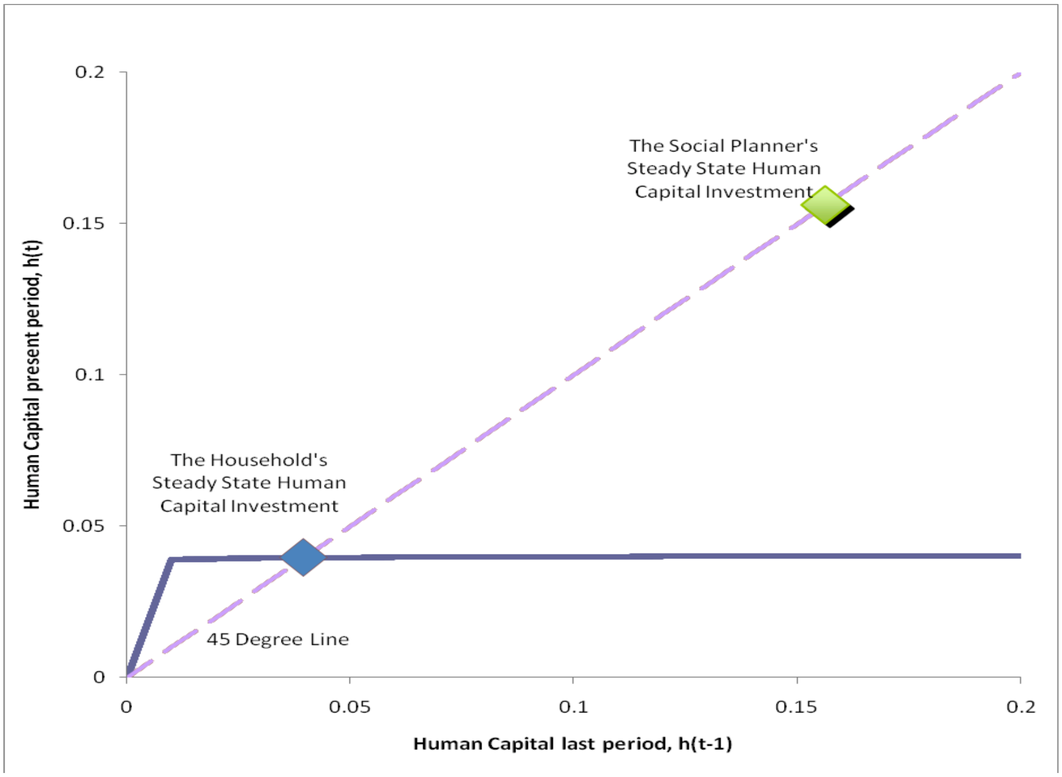

Figure 20 Household and social planner's steady state human capital investment

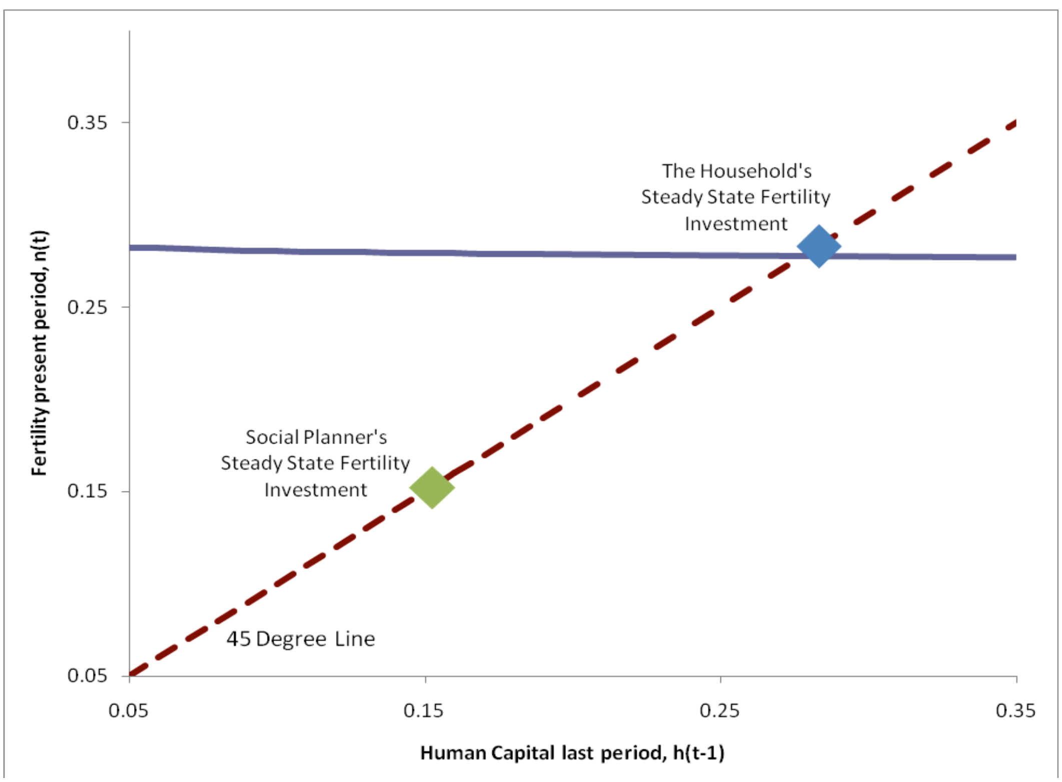

Figure 21 Household and social planner's steady state fertility investment

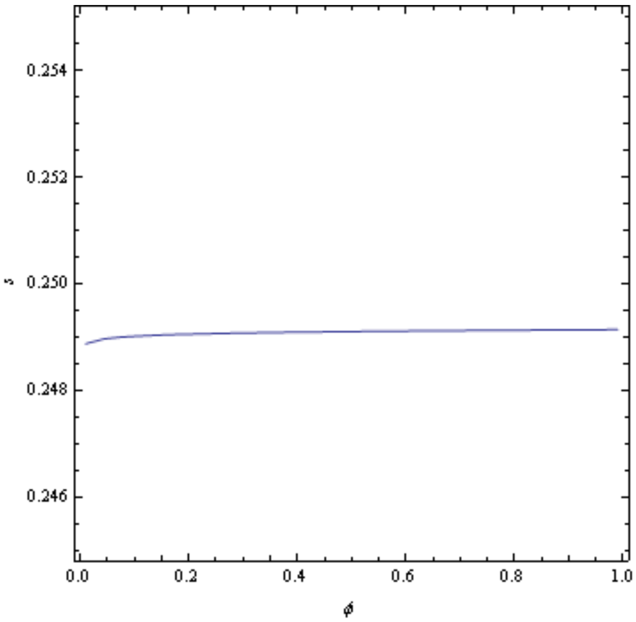

Figure 22 **Contour plot of education subsidy *s* versus opportunity cost ϕ after tax internalization**

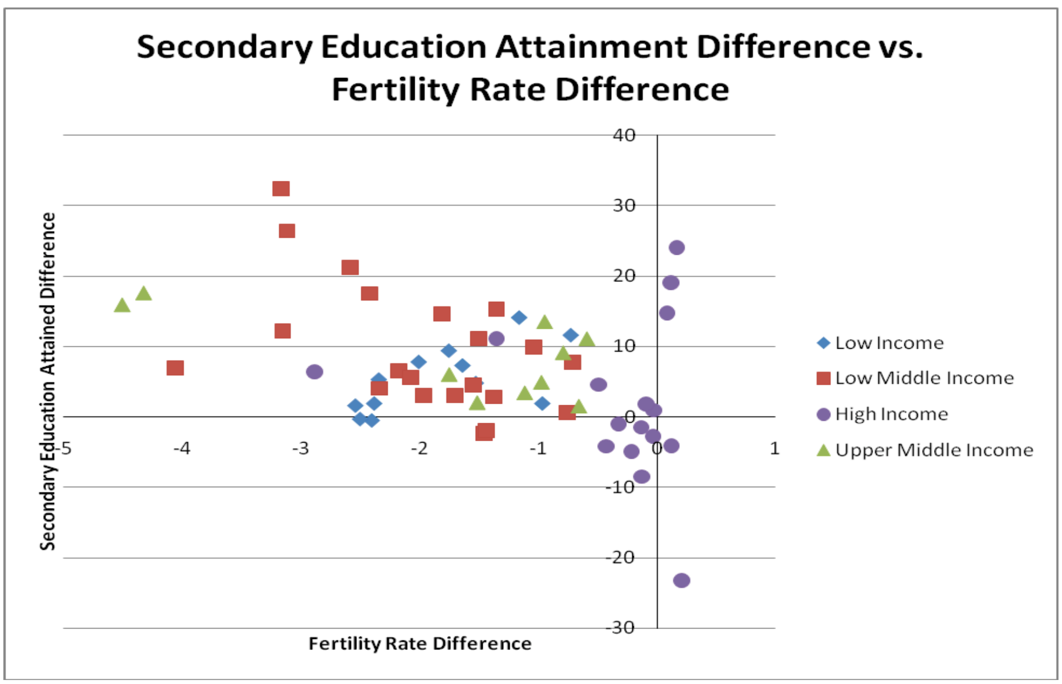

Figure 23 Difference in Secondary Education Attainment versus difference in Fertility Rate

	FERT_DIFF	LEXP_DIFF	LS_DIFF	SUB_DIFF
Mean	-1.940158	5.157289	8.365116	273.0599
Median	-1.716000	6.380000	6.900000	59.76796
Maximum	-0.590000	14.78873	32.40000	2811.672
Minimum	-4.578000	-17.34010	-2.400000	-532.5584
Std. Dev.	1.014193	7.463565	7.395748	710.5654
Skewness	-0.945348	-1.390678	1.113529	2.417271
Kurtosis	3.528671	4.384861	4.417174	8.595175
Jarque-Bera	6.905482	17.29636	12.48463	97.96614
Probability	0.031659	0.000175	0.001945	0.000000
Sum	-83.42678	221.7634	359.7000	11741.58
Sum Sq. Dev.	43.20071	2339.602	2297.278	21205936
Observations	43	43	43	43

Figure 14 Descriptive statistics for independent variables in regression

Covariance Analysis: Ordinary
Date: 04/28/09 Time: 10:41
Sample (adjusted): 1 43
Included observations: 43 after adjustments
Balanced sample (listwise missing value deletion)

Correlation	FERT_DIFF	FERT_POL	L	LEXP_DIFF	LM	LS_DIFF	SUB_DIFF	UM
FERT_DIFF	1.000000							
FERT_POL	-0.263730	1.000000						
L	0.018937	0.164384	1.000000					
LEXP_DIFF	0.078634	0.021814	-0.122944	1.000000				
LM	-0.119023	0.164859	-0.607866	-0.019171	1.000000			
LS_DIFF	-0.317154	0.048495	-0.251685	-0.493681	0.159972	1.000000		
SUB_DIFF	0.258366	-0.050233	-0.285185	0.096095	-0.106064	0.087297	1.000000	
UM	0.120724	-0.369595	-0.342494	0.153216	-0.537825	0.077940	0.428286	1.000000

Figure 24 Correlation table between regressors